Ph.D. CORE EXAM
August 2020

This exam is designed to test your broad knowledge of microeconomics and macroeconomics. You must complete all four problems during the allotted time.

IMPORTANT. You are expected to adhere to the following guidelines in completing the exam for your answer to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number all pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number.**
- Provide clear, concise discussion to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Explain your steps in any mathematical derivations. Simplify your final answers completely.
- When you turn in your exam answers double check to make sure you have included all the pages to each question number, and in order. The pages you submit as your answer are the only ones that will be considered.
- To simplify copying, please leave 1 inch borders.

Question 1

Consider a consumer, Adam, with the following utility function:

$$u(x_1, x_2) = \frac{1}{2}\ln x_1 + \frac{1}{2}\ln x_2$$

Adam's total income is m. The prices for the two goods are p_1 and p_2, respectively.

a) Find the Marshallian demand functions for goods 1 and 2. Find the indirect utility function.

b) Find the expenditure function using duality. Find the Hicksian demands using Shephard's lemma.

c) Suppose that the government imposes a sales tax on x_2 at a rate t. In other words, for every unit of x_2 purchased, the tax liability is t*p_2. Find the Marshallian demand functions for goods 1 and 2. If t increases, does the consumption of goods 1 increase, decrease, or stay the same? Does the consumption of goods 2 increase, decrease, or stay the same?

d) If Adam wants to keep his utility level the same as before the tax was enacted, how much additional income must he make?

Question 2

Sales Taxes in an Optimal Growth Model

Consider the Ramsey model of an economy in competitive equilibrium with technological growth. There is a representative household and a representative firm. The household's utility functional is

$$U \equiv \int_0^\infty u(c_t)e^{-\rho t}dt,$$

with

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta},$$

where there is no population growth, and $\rho > 0$. The representative firm has a production function $F[K_t, A_t, L_t] = K_t^\alpha (A_t L_t)^{1-\alpha}$. For simplicity, assume capital does not depreciate after production ($\delta = 0$). Further,

$$\dot{A}_t = gA_t$$

where A_t is the level of technology that grows at the constant rate g.

At every point in time, the government institutes a consumption tax (aka sales tax). That is, for every unit of consumption that it chooses, the household must pay an amount τ to the government. Find the competitive equilibrium of this economy, using the following steps.

a) Write down representative household's maximization problem, solve it, and derive the 4 equations that characterize the solution. Explain in words, intuitively, what the Hamiltonian function means, and what the 4 equations represent. Does τ show up here? Explain why or why not.

b) Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form. Derive the 2 equations that characterize the solution. Does τ show up here? Explain why or why not.

c) What are the equilibrium conditions for this economy? Does τ show up here? Explain why or why not? Derive the government budget constraint.

d) Combine your answers to parts a) - c) and derive a pair of differential equations for the variables c and k. Can you draw a phase diagram?

e) Define $\widehat{k} = \frac{k}{A}, \widehat{y} = \frac{y}{A}$, and $\widehat{c} = \frac{c}{A}$. Derive a pair of differential equations for the variables $\widehat{c}$ and $\widehat{k}$. Draw the phase diagram, carefully identifying (and deriving mathematically) all the important points.

f) What is the growth rate of the economy? What are the growth rates of consumption and capital? Discuss. Draw the phase diagram, carefully identifying (and deriving mathematically) all the important points.

g) Do the following comparative dynamics exercise: $\tau' > \tau = 0$. That is, compare the economy with and without a sales tax. As usual, the baseline economy starts in its steady state at time $t = 0$. The modified economy starts at time $t = 0$. Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of c and k for both cases. Carefully discuss your results. In particular, how does the tax affect the consumption/savings decision? Why?

Question 3

Consider a 2×2 exchange economy. Consumer 1's utility function is $U_1(x_1, y_1) = x_1 - \frac{4}{y_1} + 3$ and her endowment is $\omega_1 = (4, 0)$. Consumer 2's utility function is $U_2(x_2, y_2) = \ln(x_2) + y_2$ and his endowment is $\omega_2 = (0, 2)$.

a) Derive the contract curve (the set of Pareto optimal allocations) for this economy (please specify the entire set of points that make up the contract curve).

b) Draw an Edgeworth box showing the initial allocation, the contract curve, and one indifference curve that goes through the point $(x_1, y_1) = (3, 1)$ for consumer 1 and one indifference curve through the point $(x_2, y_2) = (1, 1)$ for consumer 2 (be careful and accurate with the Edgeworth box but the indifference curves do not need to be perfect).

c) Set up the optimization problem for each consumer and derive the offer curves (demand functions for x_1, y_1, x_2, y_2).

d) Find the competitive equilibrium for this economy. This will be a price vector $p^\star$ together with an allocation $(x^\star, y^\star)$. Confirm that this outcome is Pareto optimal, and show it on your figure.

Question 4

There are two firms that produce a good in a market. Each firm produces quantity q_i and faces inverse market demand: $P(Q) = 9 - bQ$, where $Q = q_1 + q_2$, and b is a strictly positive parameter. Firm 1 has marginal cost of production $c_1 = 1$, and firm 2 has marginal cost of production $c_2 = 2$, and neither firm has any fixed costs.

a) When both firms choose their quantity simultaneously, solve for the Nash equilibrium strategies for each firm.

b) Using the solution from part (a), solve for the quantity for each firm, the market price, and profits for each firm.

c) Now consider a two-stage game, where in the first stage either or both firms can choose to advertise (for a fixed cost), and then in the second stage both firms make simultaneous quantity decisions. Advertising in the first stage increases the market demand for the good by reducing the value of the slope parameter b. The decision that each firm faces in the first stage is whether to advertise (A) or not advertise (N). If both firms advertise then $b = 1/2$, if only one firm advertises then $b = 3/4$, and if neither firm advertises then $b = 1$. The cost of advertising is $C_A = 3$ for each firm. In stage 2 the parameter b is known by both firms (i.e., the advertising decision is revealed), and both firms simultaneously choose their quantity. Solve for the subgame-perfect Nash equilibrium strategies (ignore any possible mixed strategies in stage 1). There is no discounting between stages.

d) Finally, assume that both firms know that firm 2 <u>will not</u> advertise but firm 1 <u>may</u> advertise, and the decision by firm 1 is not known by firm 2 (i.e., b is uncertain for firm 2 at the time of their quantity decision). Firm 1 knows if it does or does not advertise. Firm 2 believes that with probability $p = 1/2$ firm 1 will not advertise and $b = 1$, and with probability $1 - p = 1/2$ firm 1 will advertise and $b = 3/4$. Ignore the first stage decision where firm 1 is deciding whether or not to advertise (i.e., just use the probabilities p and $1 - p$ as given, and assume the advertising cost is sunk). In stage 2 each firm makes a simultaneous quantity decision. Solve for the Bayesian Nash equilibrium quantity choices.