Ph.D. Core Exam -- Macroeconomics **13 August 2018 -- 8:00 am to 3:00 pm**

Part A: Answer Question A1 (required) and Question A2 or A3 (choice).

A1 (required): Short-Run Stabilization Policy and Economic Shocks

In response to economic shocks, policymakers often try to stabilize output in the short run. The appropriate stabilization policy depends on the type of shock, the degree of openness or capital mobility, and many other factors. Using the short-run/Keynesian IS-LM model (i.e., wages/prices are sticky or fixed; the capital stock and technology are constant), analyze the effects of random LM shocks and determine the best policy response under the following scenarios.

1. *Closed Economy*: Consider the following stochastic IS-LM model with static inflationary expectations and random LM shocks.

(1) $Y = E(Y\text{-}T,R,G)$ where $0 < E_{Y-T} < 1$, $E_R < 0$, $E_G = 1$ (IS)

(2) $M/P = L(Y,R) + \varepsilon$ where $L_Y > 0$, $L_R < 0$, $\varepsilon \sim N(\sigma, 0)$ (LM)

The variables are: Y = real output, E = aggregate expenditures, T = taxes, $R = r$ = interest rate, G = government purchases, M = nominal money supply, P = (fixed) price level, L = real money demand, and ε = random money demand shock.

a) Illustrate graphically and calculate/sign the relevant partial derivatives to show how this closed economy responds to a positive money demand shock ($\varepsilon > 0$).

b) What is the best monetary policy to stabilize output in the short run? Discuss.

2. *Small Open Economy*: Consider a stochastic Mundell-Fleming model with perfect capital mobility, static inflationary expectations, static exchange rate expectations, and random domestic LM shocks.

a) Using graphical analysis, explain carefully how/why this small open economy responds to a positive domestic money demand shock ($\varepsilon > 0$) under both fixed and flexible exchange rates.

b) What is the best exchange rate policy to stabilize domestic output in the short run? Discuss.

c) Does your answer to (b) change if capital mobility is very limited or zero? Discuss.

A2 (choice): Technological Change

Technology plays a prominent role in modern economies. You are asked to analyze the effects of an exogenous change in technology in the context of the following models.

1. *Classical/Long-Run Model*: Consider the following aggregate supply/demand model where wages and prices are flexible, inflationary expectations are static, the capital stock is fixed, and technology is Hicks-neutral (i.e., total factor augmenting).

(1) $W/P = A \cdot F_N(N,K)$	where $A > 0$, $F_{NN} < 0$, $F_{NK} > 0$	(labor demand)
(2) $N = NS(W/P)$	where $NS_{W/P} > 0$	(labor market equilibrium)
(3) $Y = A \cdot F(N,K)$	where $F_N > 0$, $F_K > 0$	(production function)
(4) $Y = E(Y\text{-}T,R,G)$	where $0 < E_{Y-T} < 1$, $E_R < 0$, $E_G = 1$	(IS)
(5) $M/P = L(Y,R)$	where $L_Y > 0$, $L_R < 0$	(LM)

The variables are: W/P = real wage, A = (exogenous) technology, N = labor, K = (fixed) capital stock, Y = real output, E = aggregate expenditures, T = taxes, R = r = interest rate, G = government purchases, M = nominal money supply, P = price level, and L = real money demand.

Suppose there is an exogenous increase in the level of technology.

a) Illustrate graphically how the endogenous variables are affected.

b) Explain carefully how/why the endogenous variables are affected.

2. *Solow Growth Model*: Consider the following Solow model with Hicks-neutral technology.

(1) $Y = A \cdot K^a L^{1-a}$	where $A > 0$, $0 < a < 1$	(production function)
(2) $dL/dt = nL$	where $n > 0$	(labor accumulation)
(3) $dK/dt = sY - \delta K$	where $0 < s < 1$, $0 < \delta < 1$	(capital accumulation)

The variables are: Y = total output, A = (exogenous) technology, K = capital stock, L = labor force, a = income share of capital, n = population growth rate, s = saving rate, and δ = depreciation rate.

a) Assume that technology is constant at some given level A_0. Illustrate the steady-state equilibrium in a Solow graph in (k, y)-space and derive the growth rate of total output on the balanced growth path.

b) Now suppose there is an exogenous increase in the level of technology. Illustrate graphically and explain carefully how/why the level and the growth rate of output per worker are affected over time.

A3 (choice): Statements

Select <u>any three</u> of the following statements and explain carefully why each is true, false, or uncertain in all its parts. You must use graphical and/or mathematical analysis to support your arguments. Your score depends on the quality and completeness of your explanations.

a) In the Solow growth model with human capital, where $H = L \cdot e^{\psi E}$ with $\psi > 0$ and E = years of education, a ceteris paribus decrease in the return to education (ψ) has only temporary effects on the level of output per worker and the economy's balanced growth path.

b) In the R&D growth model, there will be permanent level effects and temporary growth rate effects on output per worker when the population growth rate falls, ceteris paribus.

c) In the standard classical model, the neutrality/superneutrality of money depends solely on the absence or presence of wealth effects in the goods market.

d) In the Barro-Gordon model, the time-consistent equilibrium inflation rate will be lower with a lower natural rate of unemployment and/or a smaller impact of surprise inflation on unemployment.

Part B: Answer Both Questions.

B1: Social Security in an OLG Model

Consider an economy consisting of an infinite sequence of two period lived, overlapping generations. N_t agents are born in period t, with $n = 0$. In each period there is a single final good that is produced using a constant returns to scale technology with capital and labor as inputs. Let k_t denote the time t capital-labor ratio, and let $f(k_t)$ denote the intensive production function. Let f have the Cobb-Douglas form $f(k_t) = Ak_t^{\alpha}$, with $0 < \alpha < 1$. One unit of the final good that is not consumed at t converts into one unit of capital at $t + 1$. Capital fully depreciates after production ($\delta = 1$). Agents have the utility function

$$u(c_{1,t}, c_{2,t+1}) = \frac{c_{1,t}^{1-\theta} - 1}{1 - \theta} + \frac{c_{2,t+1}^{1-\theta} - 1}{1 - \theta}$$

with $\theta \to 1$ (log utility), and $\rho = 0$ (no time discount factor).

The government gives lump-sum social security retirement payments of σ_{t+1} in the second period of life of agents born in period t. In order to balance the budget, the government imposes a tax τ_{t+1} on labor income w_{t+1} of agents born in period *t+1* (agents born in period t get taxed τ_t when young).

a) Write down the household's maximization problem and derive the equations that characterize the solution. Discuss.

b) The firm's first-order conditions for this problem are (I am giving them to you to save you time):

$$\alpha A k_{t+1}^{\alpha-1} = R_{t+1}$$

$$(1 - \alpha) A k_t^{\alpha} = w_t$$

c) What are the equilibrium conditions for this economy? What is the government budget constraint? Explain it in words.

d) Combine your answers to parts a) - c) and derive a *Law of Motion* (*LoM*) equation that defines a difference equation for the variable k. Don't forget to use all equilibrium conditions from c). Looking at it, can we say anything about a steady-state solution? Can you graph the *LoM?*

e) Is the non-trivial steady-state in the Competitive Equilibrium (CE) Pareto Optimal (PO)? Carefully show and explain why, or why not. Under what conditions will the CE be PO? Can you find an optimal tax, so that the CE is PO?

f) Do the following comparative dynamics exercise. Initially, the CE economy is with $\sigma = \tau = 0$, and now the government imposes the optimal tax and subsidy rates that you found in part e). Draw (i) *LoM* for both cases, indicating what is different, and (ii) the time paths of the logs of c and k for both cases. Discuss.

B2: Externalities, Taxes and Subsidies in the Ak Model

Consider the model of an economy in competitive equilibrium, where there are capital externalities. There is a representative household and a representative firm. The household's utility functional is

$$U \equiv \int_0^\infty u(c_t)e^{-\rho t}dt,$$

with

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta},$$

where $1 > \rho > n = 0$, and $\theta \to 1$.

The representative firm has a production function $F\left[K_t, \bar{K}_t, L_t\right] = K_t^\alpha(\bar{K}_t L_t)^{1-\alpha}$, where $\bar{K}$ is the total quantity of capital in the economy. Normalize $L = 1$, and assume capital does not depreciate after production ($\delta = 0$). For each unit of capital the firm uses at time t, the government gives them a subsidy of σ_t. In order to balance the budget, the government imposes a tax τ_t on labor income w_t.

a) Write down representative household's maximization problem, solve it, and derive the equations that characterize the solution.

b) Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form. Derive the equations that characterize the solution.

c) What are the equilibrium conditions for this economy? What is the government budget constraint?

d) Combine your answers to parts a) - c) and derive a pair of differential equations for the variables c and k. Can you draw a phase diagram? If yes, draw the phase diagram, carefully identifying (and deriving mathematically) all the important points. Is there a balanced growth path? Show it on the graph, and derive its slope.

e) What is the growth rate of the economy? What about transitional dynamics?

f) Is the Competitive Equilibrium (CE) Pareto Optimal (PO)? If yes, why? If not, can we choose optimal subsidy and tax rates, so that the CE becomes PO?

g) Do the following comparative dynamics exercise. Initially, the CE economy is with $\sigma = \tau = 0$, and now the government introduces the optimal subsidy and tax rates that you found in part f). Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of the logs of c and k for both cases. Discuss.