Ph.D. MICROECONOMICS CORE EXAM
August 2019

This exam is designed to test your broad knowledge of microeconomics. There are three sections: one required and two choice sections. You must complete both problems in the required section and one choice problem in each of the two choice sections, giving you a total of four problems to complete during the allotted time. The required problems are in section A and the choice problems are in sections B and C. If you should answer more than one choice question in a section, only the first will be considered.

IMPORTANT. You are expected to adhere to the following guidelines in completing the exam for your answer to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number all pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number.**
- Provide clear, concise discussion to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Explain your steps in any mathematical derivations. Simplify your final answers completely.
- When you turn in your exam answers double check to make sure you have included all the pages to each question number, and in order. The pages you submit as your answer are the only ones that will be considered.
- To simplify copying, please leave 1-inch borders.

PART A: REQUIRED QUESTIONS

Both problems in Part A (A1 and A2) are required. Answer all parts of all questions.

QUESTION A1

Consider the following utility function:

$$u(x_1, x_2) = 2x_1^{1/2} + 4x_2^{1/2}$$

Let prices be denoted by p_1 and p_2 and money income be denoted by m.

a) Find the Marshallian demand functions for goods 1 and 2.

b) Find the Hicksian demand functions for goods 1 and 2.

c) Find the expenditure function and mathematically verify that Shepard's Lemma holds.

d) Find the indirect utility function and mathematically verify Roy's Identity.

QUESTION A2

Consider a 2×2 exchange economy. Consumer 1's utility function is $U_1(x_1, y_1) = x_1 + \ln(x_1) + y_1$ and her endowment is $\omega_1 = (0, 2)$. Consumer 2's utility function is $U_2(x_2, y_2) = x_2 y_2$ and his endowment is $\omega_2 = (4, 4)$.

a) Derive the contract curve (the set of Pareto optimal allocations) for this economy. Draw an Edgeworth box showing the initial allocation, the contract curve, and at least one (representative) indifference curve for each consumer.

b) Set up the optimization problem for each consumer and derive the offer curves (demand functions for x_1, y_1, x_2, y_2).

c) Find the competitive equilibrium for this economy. This will be a price vector $p^\star$ together with an allocation $(x^\star, y^\star)$. Confirm that this outcome is Pareto optimal.

d) Now suppose there is a new allocation: $\omega_1 = \left(\frac{2}{3}, 0\right)$, $\omega_2 = \left(\frac{10}{3}, 6\right)$. Find the competitive equilibrium with this new allocation (price vector and allocation).

PART B: CHOICE QUESTIONS

Answer all parts of either question B1 or B2. If you complete more than one problem, only B1 will be considered.

QUESTION B1

Consider a market with two firms with the same cost function, $c(q_i) = 10q_i + 0.5q_i^2, i = 1,2$. The market demand is characterized by $p = 100 - (q_1 + q_2)$.

a) Find the Cournot equilibrium price. Find the profits of each firm.

b) Now suppose that the manager of firm 1 has learnt that firm 2 will set its output following a Cournot model. What is firm 1's best strategy? What is firm 1's profit level?

c) For the rest of the question, suppose that both firms can choose to advertise the product. As a result, the market demand will be characterized by $p = 100 - (q_1 + q_2) + \sqrt{a_1 + a_2}$, where a_1 and a_2 denote the spending on advertisement by firm 1 and firm 2, respectively. Assume that the two firms decide to collaborate and evenly split the total profit. Find the equilibrium price and the profits of each firm.

d) Assume that the firms have spent a_1 and a_2 as derived in part c) on advertising. Do any of the firms have an incentive to change their output? In other words, is the collaboration in part c) stable?

QUESTION B2

Consider two workers with an annual income y, $y > 0$, who face a constant tax rate t, $0 < t < 1$. Workers are responsible for reporting their annual income (y_r) to the government and paying tax based on the reported income (ty_r). The government cannot directly observe a worker's true income and thus enforces compliance through audits. The probability of audit is p, $0 < p < 1$. Assume that the government can perfectly identify a worker's true income through an audit. If a worker underreported his income and was selected for an audit, he would need to pay the evaded tax at the constant tax rate t and an additional penalty tax at a rate t_p, $0 < t_p$, for the underreported portion of his true income ($y - y_r$). Assume that a worker will choose to cheat if he is indifferent between cheating and not cheating. Worker 1's utility function is $f(i) = i$ while worker 2's utility function is $g(i)$, where i denotes annual disposable income. $g(i)$ satisfies $g'(i) > 0, g''(i) < 0$.

a) Set up the utility maximization problem for worker 1.

b) Find the value t_0 such that if $t < t_0$, worker 1 would choose not to cheat. Hint: t_0 is a function of t_p and p.

c) Assume that the tax rate t equals to the value t_0 from part b. Suppose worker 1 can collude with his employer to underreport his income. If the worker colludes with his employer, the government cannot detect tax evasion in an audit. However, the worker must pay a fixed amount, C, to the employer. What is the maximum C that worker 1 would be willing to pay to collude?

d) Proof that if the tax rate t equals to the value t_0 from part b, worker 2 will choose not to cheat.

PART C: CHOICE QUESTIONS

Answer all parts of either question C1 or C2. If you complete more than one problem, only C1 will be considered.

QUESTION C1

Consider the following game of international trade. In this game there are three players: a domestic firm, a foreign firm, and the domestic government. The government moves first by setting a per unit tariff charged to the importing foreign firm (represented by t_i). Next, the foreign and domestic firms move simultaneously, choosing quantity (the domestic firm produces q_i and the foreign firm produces e_j), and the firms produce a homogeneous product. The inverse market demand curve is equal to $P = \propto - Q_i$ (where $Q_i = q_i + e_j$) and the government's objective function is the social welfare function represented by $W_i = \frac{1}{2}{Q_i}^2 + \pi_i + t_i e_j$. Each firm's objective is to maximize profit, each firm has no fixed costs, the marginal cost for the domestic firm is equal to c_i and the cost for the foreign firm is equal to c_j. Assume that $c_i = c_j = c$.

a) Solve for the optimal tariff (t_i) and the quantity that each firm will produce ($q_i\ and\ e_j$).

b) Solve for the profit that each firm will make, the tariff revenue that the government will make and the total domestic welfare.

c) In this part assume a free trade environment, where $t_i = 0$. Solve for the profit that each firm will produce, the tariff revenue that the government will make and the total domestic welfare.

d) In this final part, think about a situation where instead both countries (county *i* and country *j*) are considering a tariff (for two different goods). For simplicity assume that each government has two action choices $A_i = (t^*, t^{FT})$, where t^* is the optimal tariff rate from part a and t^{FT} is the free trade outcome (no tariff). Could the countries benefit from binding agreement where both of them commit to not implement tariffs? Explain. (Note: in this situation each countries welfare function would also include the profit that its firm makes from exporting)

QUESTION C2

Two cryptocurrency miners have joined into a mining pool and will split the rewards from their efforts equally. The amount of effort by each person is costly in terms of electricity consumed but additional effort increases the expected rewards. Define effort by the two people as E_1 and E_2. The expected reward from their combined effort is $X = E_1 + E_2 + E_1E_2$. They will split the expected reward equally, such that the shares of X to each are: $s_1 = s_2 = 0.5$. The cost of effort for person 1 is higher (because his electricity price is high): $C_1(E_1) = E_1^2$, and the cost of effort for person 2 is low: $C_2(E_2) = \frac{1}{2}E_2^2$.

a) Determine the optimal effort from each person from the perspective of the mining pool. That is, solve for the effort levels that maximizes the combined profits. Calculate the profit that each would get when they split the reward equally (but each pays their own cost). Next, solve for the effort levels if each person works independently, such that $X_i = E_i$ (in this case they do not work together or split their rewards, rather they are working for themselves only). Calculate the profits for each person in this case.

b) The two people are trying to cooperate to get each to put forth the optimal effort levels. Assume if they cooperate they will each use the optimal effort levels found in part (a), and get their respective profits. However, each person could also defect by playing a lower effort level. For each player, calculate the effort level if they defect (do this separately for each person and assume that the other person is playing the optimal level). Calculate the profits from defecting for each person (they will not be the same for each). Fill in the payoffs for a normal form 2×2 game with the strategies cooperate and defect for each (you will also need to calculate the payoffs when both play their defect strategies). What is the pure strategy Nash equilibrium? [When either or both players defect, they still split the rewards equally.]

c) A more reasonable way to represent the mining pool is a repeated game because the rewards are a given out on a regular schedule. Assume the two people are playing an infinitely repeated game. If they both cooperate in one period then they will remain in the mining pool in the next period. If either person defects in any time period then the mining pool is dissolved in the next period and each person must work independently forever after. What are the range of discount factors that support cooperation in this infinitely repeated game for each person?

d) Finally, assume that cooperation is not possible, and each will determine his effort simultaneously in a one-shot game, and they will split the combined reward equally. Assume that both people know that person 1's cost is $C_1(E_1) = E_1^2$ (as before), but only person 2 knows his true cost: $C_2(E_2) = \gamma E_2^2$. The belief by person 1 is that with equal probability γ could be ½ or 1 (i.e., $\Pr\left(\gamma = \frac{1}{2}\right) = \frac{1}{2}$ and $\Pr(\gamma = 1) = \frac{1}{2}$). Solve for the Bayesian Nash equilibrium effort levels.