Ph.D. MICROECONOMICS CORE EXAM
January 2018

This exam is designed to test your broad knowledge of microeconomics. There are three sections: one required and two choice sections. You must complete <u>both</u> problems in the required section and <u>one</u> choice problem in each of the two choice sections, giving you a total of <u>four</u> problems to complete during the allotted time. The required problems are in section A and the choice problems are in sections B and C. If you should answer more than one choice question in a section, only the first will be considered.

IMPORTANT. You are expected to adhere to the following guidelines in completing the exam for your answer to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **<u>Number all pages and organize your answers to questions in the same order as they were given to you in the exam.</u> <u>Begin your answer to each question on a new page and identify the question number.</u>**

- Provide clear, concise discussion to your answers.

- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.

- Define any notation you use in a problem and label all graphs completely.

- Explain your steps in any mathematical derivations. Simplify your final answers completely.

- When you turn in your exam answers double check to make sure you have included all the pages to each question number, and in order. The pages you submit as your answer are the only ones that will be considered.

- To simplify copying, please leave 1 inch borders.

PART A: REQUIRED QUESTIONS

Both problems in Part A (A1 and A2) are <u>required</u>. Answer all parts of all questions.

QUESTION A1

Bubba consumes only two goods: beer (B) and Goo-Goo candy (G). He is especially fond of Goo-Goos (as is anyone who has ever tried them), and so values them more in his utility function:

$$U_{Bubba} = AB^{0.2}G^{0.8}$$

where A is a constant term. Bubba earns M_{BUB} dollars per week. The price of his favorite beer, *Quittin' Time*, is P_{QT} per 6-pack (the unit of consumption), and Goo-Goos are priced at P_G.

a) Setup Bubba's utility maximization problem.

b) Derive Bubba's demand for B and G. What proportion of his income does he spend on Goo-Goos?

c) Derive Bubba's indirect utility and expenditure functions. Then, derive the compensated demand for Goo-Goos.

d) Are Goo-Goos a normal good? Are beer and Goo-Goos net substitutes or net complements? Are they gross substitutes or gross complements?

QUESTION A2

Consider an industry in which n firms produce goods that are perfect substitutes. The cost function for firm i, i $= 1,2, \dots$ n, is given by $C(q_i) = cq_i$ where q_i is firm i's output and $c > 0$. The firms are quantity setters.

The market demand is given by $p = \alpha - \beta \sum_{i=1}^{n} q_i$, where p is the market price and $\alpha, \beta > 0$. Assume that $\alpha > c$. Assume that this is an infinite repeated game and that all firms have a common discount factor δ, $0 < \delta < 1$.

a) Find the pure strategy Nash equilibrium of this game.

b) Suppose that the firms form a collusive agreement that they produce equal outputs and split profits evenly. What is the maximum profit per firm can be obtained?

c) Suppose $n = 2$ and $\delta = 0.75$. Can the profit per firm you found in b) be supported in a subgame perfect equilibrium of the infinite repeated game? What is the minimum δ for $n = 2$, for which this can be done?

d) Now suppose that δ is fixed. Show that as $n \to \infty$ the per firm profits from b) cannot be supported in any subgame perfect Nash equilibrium.

PART B: CHOICE QUESTIONS

Answer all parts of either question B1 _or_ B2. If you complete more than one problem, only B1 will be considered.

QUESTION B1

Suppose the estimated ordinary demand function for a public water utility (for which there is no substitute) is given by:

$$w = 75 - 10P$$

where w is the number of units per period and P is the price per unit. The price per unit is $2.00 (set by the utility's oversight board). The typical (average) consumer spends 0.5% of his or her income on water and his or her income elasticity of demand for water is 0.2.

a) Find the compensated elasticity of demand at the current price for the average consumer.

b) The local municipality, whose objective is to maximize societal welfare, currently runs the utility. Due to maintenance problems in the system, the water system is constrained at 50,000 units of water per period. Suppose there are 1000 identical consumers and suppose the marginal cost per unit of water is $2.00 and there is a fixed cost of $10,000 per period. What is the optimal price the utility will charge and how much water will be used?

c) The municipality is considering selling the utility to a for profit firm. Given the circumstances in b), what is the optimal price the firm will set and how much water will be consumed?

d) If the utility is sold, what is the total amount of compensation that would need to be given to the average consumer to make them as well off as they were prior to the sale?

QUESTION B2

Frank owns a company that manufactures widgets (q) using inputs x_1 and x_2 according to the production function $q = \frac{1}{\frac{1}{x_1}+\frac{1}{x_2}}$. The cost function that Frank faces is given by $C = w_1 x_1 + w_2 x_2$, where w_1 is the per unit cost of x_1 and w_2 is the per unit cost of x_2.

a) Show that this production function belongs to the CES family and calculate the elasticity of substitution.

b) Find the conditional input demand functions for x_1 and x_2 when $q = 1$. Then, use this to calculate the cost function as a function of only w_1, w_2, and constants.

c) Let $x_1(w_1, w_2, 1)$ and $x_2(w_1, w_2, 1)$ be the conditional input demands you solved for in part (b). Define the ratio of "factor shares" as: $\frac{w_1 x_1(w_1,w_2,1)}{w_2 x_2(w_1,w_2,1)}$. Then, how does the ratio of factor shares vary with the ratio w_1/w_2?

d) Assume that the quantity of input x_1 is fixed in the short-run at 10 units. Solve for the firm's short-run expansion path. In words, explain using economic intuition why the expansion path has the slope it does.

PART C: CHOICE QUESTIONS

Answer all parts of either question C1 _or_ C2. If you complete more than one problem, only C1 will be considered.

QUESTION C1

The population consist of two type of agents, denoted types 1 and 2. All agents have the same utility function: $U(W) = \sqrt{W}$ with W denoting wealth. A proportion p of the population is of type 1 and a proportion $(1 - p)$ is of type 2. Type 1 agents have a probability of accident equal to 0.5, while type 2 agents have a probability of accident equal to 0.25. If an accident occurs, one suffers a loss of $72. All events are independently distributed. All agents have a wealth endowment of $100.

There are two risk neutral insurance firms whose only cost of providing insurance is the payout in case of an accident. We will study pure strategy subgame perfect Nash equilibria of the following game:

Stage 1: each firm simultaneously announces a set of contracts. A contract is a pair (a, b) where a is the payment from the firm to the agent if an accident occurs and b is the payment from the agent to the firm if an accident does not occur.

Stage 2: given the contracts offered, each agent chooses whether to accept any contract and if so which one. If any agent is indifferent between two contacts, she randomizes.

a) Assume that the type of an agent is observable. Find the equilibrium contract for each group.

b) For the rest of the question, assume that agent type is not observable. Is the contract structure you found in a) an equilibrium? Explain your answer.

c) Can you find an equilibrium involving just one common contract offered by each firm? Explain your answer.

d) Assume that an equilibrium exists. Find the contract that is purchased by type 1 agents. Characterize the contract that is purchased by type 2 agents (you do not need to explicitly solve for the contract that is purchased by type 2 agents).

QUESTION C2

Wall-E-Mart is currently the only retailer of waste disposal systems in the town of Futuredale. Wall-E-Mart buys waste disposal systems from a wholesale company for c dollars per unit. The inverse demand curve for waste disposal systems in Futuredale is

$$P(q) = a - bq$$

for $q \leq \frac{a}{b}$, and 0 otherwise, where P is the price and q is the quantity of waste disposal systems.

a) Solve for the profit maximizing quantity and price of Wall-E-Mart when it acts as a monopolist. Solve for the profit for Wall-E-Mart.

b) Now suppose the residents of Futuredale form a Consumer Union that bargains directly with Wall-E-Mart. The goal of the Consumer Union is the maximize consumer surplus for residents of Futuredale. Suppose that the Consumer Union gets to make a take-it-or-leave-it offer to Wall-E-Mart that specifies the quantity (q) the Consumer Union will purchase and the total amount they will pay (R). Wall-E-Mart can then either accept or reject the offer. If the offer is accepted, trade occurs as specified in the offer. If the offer is rejected, the situation will revert to monopoly and Wall-E-Mart will earn monopoly profits as in part (a). Find the subgame perfect equilibrium.

c) Now suppose that there is no union but there is a potential entrant, Bulls-Eye Corporation, who is considering opening a store that sells waste disposal systems in Futuredale. Bulls-Eye Corporation can buy waste disposal systems from a wholesale company at a cost of c dollars per unit. Bulls-Eye Corporation has a fixed costs of entry of F dollars. Solve for the Cournot equilibrium level of outputs for each firm (q_W, q_B) if Bulls-Eye Corporation enters. Under what conditions will it be profitable for Bulls-Eye Corporation to enter?

d) Suppose the parameter values are $a = 9, b = 1, c = 2, F = 2$, and there is both a Consumer Union and a potential entrant, Bulls-Eye Corporation. As in part (b), the Consumer Union will present a take-it-or-leave-it offer to Wall-E-Mart, specifying total quantity and total revenue R. Wall-E-Mart must either accept or reject the offer. In either case, Bulls-Eye will have the option to enter the market. If Wall-E-Mart accepts the offer: either (1) Bulls-Eye enters (and pays F) and Bulls-Eye and Wall-E-Mart split R in half, and each produces half of the required quantity or (2) Bulls-Eye stays out and Wall-E-Mart gets the entire R and produces the entire quantity. If Wall-E-Mart rejects the offer: either (1) Bulls-Eye enters (and pays F) and they get Cournot outcome from part (c), or (2) Bulls-Eye stays out and Wall-E-Mart gets monopoly profits from part (a). Find the subgame perfect equilibrium, specifying the offer and the actions of each firm.