Ph.D. MICROECONOMICS CORE EXAM
October 2019

This exam is designed to test your broad knowledge of microeconomics. There are three sections: one required and two choice sections. You must complete both problems in the required section and one choice problem in each of the two choice sections, giving you a total of four problems to complete during the allotted time. The required problems are in section A and the choice problems are in sections B and C. If you should answer more than one choice question in a section, only the first will be considered.

IMPORTANT. You are expected to adhere to the following guidelines in completing the exam for your answer to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number all pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number.**
- Provide clear, concise discussion to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Explain your steps in any mathematical derivations. Simplify your final answers completely.
- When you turn in your exam answers double check to make sure you have included all the pages to each question number, and in order. The pages you submit as your answer are the only ones that will be considered.
- To simplify copying, please leave 1 inch borders.

PART A: REQUIRED QUESTIONS

Both problems in Part A (A1 and A2) are required. Answer all parts of all questions.

QUESTION A1

Consider the following utility function:

$$u(x_1, x_2) = \frac{1}{\frac{1}{x_1} + \frac{4}{x_2}}$$

Let prices be denoted by p_1 and p_2 and income be denoted by y.

a) Find the Marshallian demand functions for goods 1 and 2.

b) Find the consumer's indirect utility function. Verify Roy's identity.

c) Find this consumer's expenditure function using duality.

d) Find the Hicksian demand functions for goods 1 and 2.

QUESTION A2

Consider a 2×2 exchange economy. Consumer 1's utility function is $U_1(x_1, y_1) = x_1 + 2\ln(y_1)$ and her endowment is $\omega_1 = (4, 2)$. Consumer 2's utility function is $U_2(x_2, y_2) = 4\ln(x_2) + 2y_2$ and his endowment is $\omega_2 = (4, 4)$.

a) Derive the contract curve (the set of Pareto optimal allocations) for this economy (please specify the entire set of points that make up the contract curve).

b) Draw an Edgeworth box showing the initial allocation, the contract curve, and an indifference curve that goes through the initial endowment (be careful and accurate with the Edgeworth box).

c) Set up the optimization problem for each consumer and derive the offer curves (demand functions for x_1, y_1, x_2, y_2).

d) Draw the offer curves from part c) on the Edgeworth box. Plot at least three points (i.e., choose at least three price ratios) on each consumer's offer curve. Find the competitive equilibrium for this economy. This will be a price vector $p^\star$ together with an allocation $(x^\star, y^\star)$. Confirm that this outcome is Pareto optimal, and show it on your figure.

PART B: CHOICE QUESTIONS

Answer all parts of either question B1 or B2. If you complete more than one problem, only B1 will be considered.

QUESTION B1

Suppose the city of Easthallow is operating a lottery game. The outcome of each lottery ticket is independent. The payment schedule is as follows:

Probability	Payment
0.5	$0
0.3	$1
0.15	$2
0.05	$8

Adam is endowed with $10. His utility function is $u(w) = w^{0.5}$, where w denotes his wealth.

a) Is Adam risk averse, risk neutral, or risk loving? Find Adam's Arrow-Pratt measure of absolute risk aversion.

b) Find the "actuarially fair" price of the lottery ticket. In other words, find the price of the lottery ticket such that the lottery company's profit is zero. Will Adam buy a lottery ticket at this price? Why? Provide a detailed mathematical justification.

c) Suppose a friend gave Adam a lottery tickets for his birthday. Will Adam be willing to sell the lottery ticket at the "actuarially fair" price? Why? Provide a detailed mathematical justification.

d) Adam's friend, Bob, is also endowed with $10. Bob's utility function is $u(w) = \ln(w)$. Does there exist a price, p, such that Bob will be willing to buy the lottery ticket from Adam, and Adam will be willing to sell? You don't need to find a specific range for p. Rather, write out the condition(s) that it must satisfy.

QUESTION B2

Fred the farmer grows and sells New Mexico Hatch green chile (he refuses to grow that inferior Colorado-made Pueblo chile). Demand for Hatch green chile is:

$$D(p) = 500 - 15p$$

Fred's cost to grow and sell the chile is given by:

$$C(q) = 5 + 2q^2 - 4q$$

where q is the quantity of green chile grown and sold.

a) Due to a drought last summer, the supply of Hatch green chile is fixed in the short-run at 400 units. Assuming Fred the farmer is a profit maximizer, and further assuming a competitive market, find the equilibrium price of green chile. How many green chiles will he sell? How much profit will Fred earn?

b) If the short-run supply of green chile remains fixed at 400 units (and all other model parameters remain fixed over time), will there be entry or exit into the Hatch green chile industry in New Mexico? Explain in words. Assume that all green chile farmers in NM have identical cost structures as Fred. Then, find what the new long-run supply <u>and</u> equilibrium price will be after new farmers enter (if you think entry will occur) or after some farmers exit (if you think exit will occur).

c) Suppose that a wildfire destroys all this year's crop of Hatch chile except for Fred's crop and his neighbor (and competitor) Chuck's crop. Chuck's cost to grow and sell chile is given by:

$$C(q)_{Chuck} = 3 + 3q^2 - 3q$$

Assume Fred and Chuck face the same market demand and further assume that supply is no longer fixed. Sadly, no other green chile farmers were able to restart their operations after the wildfire.
Assuming a Cournot structure, what will be the price, quantities, and profits in the chile market now?

d) Fred is now considering whether he wants to merge his chile farm with Chuck's farm. Assume that Chuck is willing to sell his farm for an annual payment of $100. Because of the wildfire, if Fred buys Chuck's chile farm, he will be the sole supplier of Hatch green chile. The cost function of the combined firm is given by:

$$C(q)_{combined} = 4 + 2q^2 - 3q$$

Make a sound economic argument (using ample math) for why or why not Fred should purchase Chuck's farm versus playing the Cournot game with Chuck in part c).

PART C: CHOICE QUESTIONS

Answer all parts of either question C1 or C2. If you complete more than one problem, only C1 will be considered.

QUESTION C1

You are a homeowner (H) and your A/C has gone out in the middle of the summer. You call the repairperson (R) to fix your A/C. He identifies the problem and now you must determine whether to make the repairs and how much it will cost. The homeowner has a value, v, of having the A/C fixed, and the repairperson has a cost, c, of doing the work. Both H and R must submit a price for the job. Each can offer either a low price (L), medium price (M) or outrageous price (O) (where $O > M > L$). The price by the homeowner and the repairman are p_H and p_R, respectively. If $p_H \geq p_R$ then the job will be done at $p = \frac{p_H + p_R}{2}$, and the payoff to H is $v - p$, and the payoff to R is $p - c$. Otherwise, the job will not be done and each gets zero.

a) Assume both v and c are known by both H and R and the bids will be submitted simultaneously. Write out the normal form of this game.

b) Solve for the pure-strategy Nash equilibria from the game in part a in both of the following scenarios.
   i. $v > O$ and $c < L$
   ii. $v = M$ and $L < c < M$

c) Now suppose that the repairperson gets to make an offer first (same options as before: L, M, O) and then the homeowner responds with an offer (L, M, O). The price and payoffs are determined the same as above. Write out the extensive form of this game. Solve for all subgame perfect Nash equilibria in each of the following scenarios (remember a strategy is an action at every information set)
   i. $v > O$ and $c < L$
   ii. $v = M$ and $L < c < M$

d) Finally, go back to the simultaneous game in parts a) and b). Suppose $L = 100$, $M = 200$, $O = 400$, $v = 250$, $c = 75$. Find all pure and mixed strategy equilibria of this game.

QUESTION C2

Consider a t period, n firm, Cournot game where each firm's objective is to maximize profit, and the industry has an inverse demand curve equal to $P(Q) = \alpha - Q$ (where of course, $Q = q_1 + ... + q_n$). Each firm as a has a cost function equal to $C(q_i) = c_i q_i$ (where $c_i < \alpha$, for all i), and a discount rate equal to δ.

a) First, assume $t = n = 1$. What quantity will the firm produce and what will the market price be? What is the consumer surplus, and what is the deadweight loss compared to a perfectly competitive industry?

b) Next, assume that $t = 1, n = 2$, and $c_1 = 2c_2$. Describe the Nash equilibria for this game. How much profit will each firm make? (solve for profit in terms of α and c_2)

c) Now assume that this is an infinitely repeated game with n firms, and each firm has an identical cost structure ($c_i = c$ for all i). What is the lowest value of δ such that firms can use trigger strategies to sustain the cartel output level (firms split optimal profits evenly) in a subgame-perfect Nash equilibrium? Be sure to state the strategies that each firm plays and describe your notation.

d) Explain what a subgame is and provide a definition of a subgame-perfect Nash Equilibrium. Why do subgame-perfect Nash equilibria sometimes provide more reasonable predictions to game-theoretic problems? Consider the following strategy: play cartel output (firms split optimal profits evenly) in period t, if the other player played cartel output in period t, play cartel output in period t+1, otherwise play $q_{t+1} = \infty$.Would this strategy be a reasonable strategy to play? If each player played this strategy would it be a Nash Equilibrium? Would this strategy combination be a subgame-perfect Nash Equilibrium? Explain.