Part B: Answer B1 (required), plus either question B2 or B3.

B1 (required): Financial Intermediation

Consider the Diamond-Dybvig model with two assets. There are three periods: $t = 0, 1, 2$. Agents are ex-ante identical. They are endowed one unit of a single good at $t = 0$, and nothing at $t = 1, 2$. At the beginning of $t = 1$, a fraction π of agents learn that they prefer to consume only at $t = 1$, while the remaining fraction $(1 - \pi)$ of agents prefers to consume only at $t = 2$. There is a linear production technology whereby one unit of the good invested in period 0 yields $R = 2$ units of the good at time 2. This technology is illiquid, in the sense that an investment that is interrupted in period 1 generates $r = 0$ units of consumption. In addition, there is a liquid storage technology, whose return is equal to 1 in both periods. Agents preferences are given by

$$u(c) = ln(c) \tag{1}$$

a) Write down the problem of an agent in autarky, the FOC, and solve for the optimal consumption vector (c_1, c_2). What happens to the consumption vector when $\pi \geqslant 1/2$? When $\pi = 0$? Graph this.

b) Now suppose that in period 1, after agents learn their idiosyncratic consumption preference shock and before they consume, a financial market opens where agents can trade claims for the returns on the illiquid production technology. Let p be the price of a bond that yields one unit of the illiquid production technology at $t = 2$. Write down the problem of an agent in this setting. What will the equilibrium price of a bond be in this case (and why)? What is the consumption vector (c_1, c_2)? Discuss.

c) Now, instead of a financial market, suppose agents form coalitions, which they call banks, and pool their resources. Write down the problem of the bank, the FOC, and the optimal consumption vector.

d) Compare and discuss c) with a) and b).

e) Can multiple equilibria arise in this environment? Why/Why not? Carefully discuss.

B2: Unfunded Social Security in an OLG Model

Consider an economy consisting of an infinite sequence of two period lived, overlapping generations. N_t agents are born in period t, with $N_{t+1} = N_t$. In each period there is a single final good that is produced using a constant returns to scale technology with capital and labor as inputs. Let k_t denote the time t capital-labor ratio, and let $f(k_t)$ denote the intensive production function. Let f have the Cobb-Douglas form $f(k_t) = Ak_t^{\alpha}$, with $0 < \alpha < 1$. One unit of the final good that is not consumed at t converts into one unit of capital at $t+1$. Capital depreciates after production, with $\delta \in (0, 1)$. Agents have the utility function

$$u(c_{1,t}, c_{2,t+1}) = \frac{c_{1,t}^{1-\theta} - 1}{1-\theta} + (1+\rho)^{-1} \frac{c_{2,t+1}^{1-\theta} - 1}{1-\theta}$$

with $\theta \to 1$.

a) Write down the household's maximization problem and derive the equations that characterize the solution $(a_t, c_{1,t}, c_{2,t+1})$. Discuss.

b) Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form.

c) What are the equilibrium conditions for this economy?

d) Combine your answers to parts (a) - (c) and derive a *Law of Motion* (*LoM*) equation that defines a difference equation for the variable k. Looking at it, can we say anything about a steady-state solution? Can you graph the *LoM?*

e) Is the non-trivial steady-state in the Competitive Equilibrium (CE) Pareto Optimal (PO)? Carefully show and explain why, or why not.

f) Now suppose that, besides saving in assets, young agents born in period t can choose how much to contribute to social security (label this variable d_t). That contribution is given by the government to the current old generation. Then, agents born in period t, receive d_{t+1} from the government when old. Will the Competitive Equilibrium (CE) in this problem be Pareto Optimal (PO)? Carefully explain why, or why not.

B3: Income Taxes in an Optimal Growth Model

Consider the Ramsey model of an economy in competitive equilibrium. There is a representative household and a representative firm. The household's utility functional is

$$U \equiv \int_0^\infty u(c_t)e^{-\rho t}dt,$$

with

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta},$$

where there is no population growth, and $\rho > 0$. The representative firm has a constant returns to scale per worker production function $f(k_t) = Ak_t^\alpha$. For simplicity, assume capital does not depreciate after production ($\delta = 0$). At every point in time, assume that the government institutes an income tax. That is, for every unit of income, the household must pay an amount τ to the government. The government then deposits the taxes in an offshore bank. Find the competitive equilibrium of this economy, using the following steps.

a) Write down representative household's maximization problem, solve it, and derive the 4 equations that characterize the solution. Explain in words, intuitively, what the Hamiltonian function means, and what the 4 equations represent.

b) Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form. Derive the 2 equations that characterize the solution.

c) What are the equilibrium conditions for this economy? Derive the government budget constraint.

d) Combine your answers to parts a) - c) and derive a pair of differential equations for the variables c and k.

e) Draw the phase diagram, carefully identifying (and deriving mathematically) all the important points.

f) Do the following comparative dynamics exercise: $\tau' > \tau = 0$. That is, compare the economy with and without the tax. As usual, the baseline economy starts in its steady state at time $t = 0$. The modified economy starts at time $t = 0$. Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of c and k for both cases. Carefully discuss your results. In particular, how does the tax affect the consumption/savings decision? Why?