
Part A: Answer Question A1 (required) and Question A2 or A3 (choice).

A1 (required): The End of Easy Money

Signaling the end of easy money, the Fed (U.S. central bank) has begun selling off some of its huge holdings of U.S. government bonds. Consider the following aggregate supply/demand model of the economy, which includes a wealth effect in the goods market. The public holds real wealth in the form of money and government bonds, the capital stock and technology are fixed, and inflationary expectations are static.

(1)	$W/P = F_N(N,K)$	where $F_{NN} < 0$, $F_{NK} > 0$	(labor demand)
(2)	$N = NS(W/P)$	where $NS_{W/P} > 0$	(labor market equilibrium)
(3)	$Y = F(N,K)$	where $F_N > 0$, $F_K > 0$	(production function)
(4)	$Y = E(Y\text{-}T,R,V,G)$	where $0 < E_{Y-T} < 1$, $E_R < 0$, $0 < E_V < 1$, $E_G = 1$	(IS)
(5)	$M/P = L(Y,R)$	where $L_Y > 0$, $L_R < 0$	(LM)

The variables are: W/P = real wage, N = labor, K = (fixed) capital stock, Y = real output/income, E = aggregate expenditures, T = taxes, R = nominal/real interest rate, G = government purchases of goods and services, $V = (M+B)/P$ = real wealth, B = government bonds held by the public, M = nominal money supply, P = price level, and L = real money demand.

Analyze the effects of the Fed's open market sale of government bonds for the following time horizons:

1. <u>*Short Run (i.e., Keynesian world with fixed wages and prices)*</u>

    a) Show graphically and explain in detail how/why the endogenous variables responds to the open market sale of government bonds.

    b) Explain how your answer to (a) changes under the following scenarios:

        i. Investment spending is perfectly interest-inelastic.

        ii. The U.S. economy is a small open economy with perfect capital mobility and flexible exchange rates that can be described by the basic Mundell-Fleming model.

2. <u>*Long Run (i.e., Classical world with flexible wages and prices)*</u>

    a) Determine how the endogenous variables respond to the open market sale of government bonds by calculating and signing the relevant derivatives using Cramer's Rule. Discuss whether money is neutral and/or superneutral here.

    b) Now suppose that labor supply is a function of both real wages and interest rates, i.e., $N = NS(W/P,R)$ with $NS_R < 0$. Show graphically and explain how/why the endogenous variables respond to the open market sale of government bonds run under these circumstances.

A2 (choice): Optimal Monetary Policy

Consider the following stochastic version of the Barro-Gordon model, where the policymaker seeks to minimize the social loss function by selecting the optimal inflation rate (π_t) directly:

(1) $y_t = y_n + b(\pi_t - \pi_t^e) + \varepsilon_t$ with $b > 0$, $\varepsilon \sim N(0, \sigma^2)$ (Lucas Supply Curve)

(2) $S_t = \pi_t^2 + (y_t - y^*)^2$ with $y^* > y_n$ (Social Loss Function)

(3) $\pi_t^e = E_{t-1}\pi_t$ (Rational Expectations)

The variables are: y = real output, y_n = natural level of output, y^* = target level of output, π = actual inflation rate, π^e = expected inflation rate, S = social loss due to deviations of inflation and output from their target levels, and E = expectations operator. The (implicit) target level of inflation is zero.

a) Illustrate graphically and explain why the zero inflation policy is time-inconsistent.

b) Calculate the time-consistent equilibrium inflation rate and the expected social loss in the discretionary equilibrium.

c) Explain intuitively and show mathematically whether society is expected to be better or worse off under the following circumstances:

   i. The natural output level increases.

   ii. Output becomes more responsive to unexpected inflation.

A3 (choice): Statements

Select <u>any three</u> of the following statements and explain carefully why each is true, false, or uncertain in all its parts. You must use graphical and/or mathematical analysis to support your arguments. Your score depends on the quality and completeness of your explanations.

a) According to the Solow growth model, countries with higher saving rates, lower population growth rates, and higher skill ratios tend to have higher levels of output per worker in the long run.

b) In the R&D growth model, there will be permanent level effects and temporary growth rate effects on output per worker when the population growth rate increases.

c) The extent of crowding out after a fiscal expansion depends solely on the degree of wage and price flexibility in the economy.

d) To minimize short-run output fluctuations caused by random IS shocks, policymakers in a closed economy should fix the interest rate while policymakers in a small open economy should fix the exchange rate.

Part B: Answer Both Questions

B1: Human Capital

Consider the Ramsey model of an economy in competitive equilibrium. There is a representative household and a representative firm. The household's utility functional is

$$U \equiv \int_0^\infty u(c_t)e^{-\rho t}dt,$$

with

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta},$$

where $1 > \rho > n = 0$, and $\theta > 0$.

The representative firm has a production function

$$F\left[K_t, H_t, L_t\right] = AK_t^{\alpha}(H_t L_t)^{1-\alpha},$$

where H is the total stock of human capital in this economy. Further assume human capital grows at a constant rate h. That is,

$$\dot{H}_t = hH_t$$

For simplicity, normalize $L = 1$, and assume capital does not depreciate after production ($\delta = 0$). Find the competitive equilibrium of this economy, using the following steps.

a) Write down the representative household's maximization problem, solve it, and derive the 4 equations that characterize the solution.

b) Write down the firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form. Derive the 2 equations that characterize the solution.

c) What are the equilibrium conditions for this economy?

d) Combine your answers to parts a) - c) and derive a pair of differential equations for the variables c and k. Can you draw a phase diagram? If so, carefully identify (and derive mathematically) all the important points. If you can't draw a phase diagram, can you transform the differential equations in order to be able to draw a phase diagram? Is there a balanced growth path? What is its slope? What is the growth rate of the economy?

e) Is the Competitive Equilibrium Pareto Optimal?

f) Do the following comparative dynamics exercise: $h' > h$. Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of the logs of c and k for both cases. Discuss.

B2: Financial Intermediation

Consider the Diamond-Dybvig model with two assets. There are three periods: $t = 0, 1, 2$. Agents are ex-ante identical. They are endowed one unit of a single good at $t = 0$, and nothing at $t = 1, 2$. At the beginning of $t = 1$, a fraction π of agents learn that they prefer to consume only at $t = 1$, while the remaining fraction $(1 - \pi)$ of agents prefers to consume only at $t = 2$. There is a linear production technology whereby one unit of the good invested in period 0 yields $R > 1$ units of the good at time 2. This technology is illiquid, in the sense that an investment that is interrupted in period 1 generates $r < 1$ units of consumption. In addition, there is a liquid storage technology, whose return is equal to 1 in both periods. Agents preferences are given by

$$u(c) = \lim_{\theta \to \infty} \frac{c_t^{1-\theta} - 1}{1 - \theta}$$

a) Write down the problem of an agent in autarky. What is the optimal consumption vector (c_1, c_2)? Explain.

b) Now suppose that in period 1, after agents learn their idiosyncratic consumption preference shock and before they consume, a financial market opens where agents can trade claims for the returns on the illiquid production technology. Let p be the price of a bond that yields one unit of the illiquid production technology at $t = 2$. What is the optimal consumption vector in this case?

c) Now suppose agents form coalitions, which they call banks, and pool their resources. Write down the problem of the bank. What is the optimal consumption vector (c_1, c_2)? Explain.

d) Carefully compare and discuss a)-c).

e) Can multiple equilibria arise in this banking environment? Why/Why not? If yes, what policies will prevent multiple equilibria (ie, prevent banking panics?). What would be the new consumption vectors under these new rules? Carefully discuss, showing new models and equations, if necessary.