Ph.D. Core Exam -- Macroeconomics **12 August 2015 -- 8:00 am to 3:00 pm**

Part A: Answer question A1 (required), plus either question A2 or A3.

A1 (required): Population Growth—Gain or Drain?

With the world population expected to grow to 9 billion by 2050, some people argue that continued population growth is a drain on standards of living and economic growth while others point to the potential gains due to increased research and development (R&D). As an economist, you are asked to sort out these arguments in the context of the following models.

1. Consider a standard Solow growth model, where total output (Y) is a constant-returns-to-scale production function of physical capital (K) and effective labor (AL).

(1)	$Y = K^{\alpha}(AL)^{1-\alpha}$	where $0 < \alpha < 1$	(production function)
(2)	$dK/dt = sY$	where $0 < s < 1$	(capital accumulation)
(3)	$dL/dt = nL$	where $n > 0$	(labor accumulation)
(4)	$dA/dt = g_A A$	where $g_A > 0$	(technical progress)

The other variables are: A = labor-augmenting technology/knowledge, L = labor force, α = income share of capital, s = saving rate, n = population growth rate, g_A = growth rate of technology.

a) Characterize the initial equilibrium for this economy by (i) showing the steady-state equilibrium in a Solow graph, (ii) calculating output per worker ($y = Y/L$) in steady-state, and (iii) deriving the growth rate of output per worker on the balanced growth path.

b) Illustrate graphically and explain how/why both the level and the growth rate of output per worker respond over time to a ceteris paribus increase in the population growth rate.

2. Now consider a simple R&D/endogenous growth model without physical capital, where a fraction a_L of the labor force is employed in the R&D sector.

(1)	$Y = A(1-a_L)L$	where $0 < a_L < 1$	(output production)
(2)	$dA/dt = (a_L L)^{\gamma} A^{\theta}$	where $\gamma > 0, \theta < 1$	(knowledge production)
(3)	$dL/dt = nL$	where $n > 0$	(labor accumulation)

a) Characterize the dynamics this economy by (i) calculating the growth rate of knowledge in steady-state and (ii) determining whether the economy is on a balanced growth path.

b) Explain how/why the growth rate of output per worker responds over time to a ceteris paribus increase in the population growth rate. Discuss how this result can be reconciled with the stylized facts of economic growth.

A3: Statements

Select any three of the following statements and explain carefully why each is true, false, or uncertain in all its parts. You must use graphical and/or mathematical analysis to support your arguments. Your score depends on the quality and completeness of your explanations.

1. According to the basic classical model with flexible wages/prices, an earthquake that destroys part of the country's capital stock will cause output, employment, real wages, and interest rates to fall while prices will rise.

2. Whether the Fisher effect is full or partial depends solely on the degree of wage/price flexibility in the economy.

3. In a small open economy with fixed wages/prices, fiscal policy is more effective the higher the degree of capital mobility.

4. According to the Barro-Gordon model, the time-consistent equilibrium inflation rate will be higher the less inflation-averse the policymaker and the greater the sensitivity of unemployment to unexpected inflation.

Part B: Answer B1 (required), plus either question B2 or B3.

B1 (required): Financial Intermediation

Consider the Diamond-Dybvig model with two assets. There are three periods: $t = 0, 1, 2$. Agents are ex-ante identical. They are endowed one unit of a single good at $t = 0$, and nothing at $t = 1, 2$. At the beginning of $t = 1$, a fraction π of agents learn that they prefer to consume only at $t = 1$, while the remaining fraction $(1 - \pi)$ of agents prefers to consume only at $t = 2$. There is a linear production technology whereby one unit of the good invested in period 0 yields $R = 2$ units of the good at time 2. This technology is illiquid, in the sense that an investment that is interrupted in period 1 generates $r = 0$ units of consumption. In addition, there is a liquid storage technology, whose return is equal to 1 in both periods. Agents preferences are given by

$$u(c) = ln(c) \tag{1}$$

a) Write down the problem of an agent in autarky, the FOC, and solve for the optimal consumption vector (c_1, c_2). What happens to the consumption vector when $\pi \geqslant 1/2$? When $\pi = 0$? Graph this.

b) Now suppose that in period 1, after agents learn their idiosyncratic consumption preference shock and before they consume, a financial market opens where agents can trade claims for the returns on the illiquid production technology. Let p be the price of a bond that yields one unit of the illiquid production technology at $t = 2$. Write down the problem of an agent in this setting. What will the equilibrium price of a bond be in this case (and why)? What is the consumption vector (c_1, c_2)? Discuss.

c) Now, instead of a financial market, suppose agents form coalitions, which they call banks, and pool their resources. Write down the problem of the bank, the FOC, and the optimal consumption vector.

d) Compare and discuss c) with a) and b).

e) Can multiple equilibria arise in this environment? Why/Why not? Carefully discuss.

B3: Income Taxes in an Optimal Growth Model

Consider the Ramsey model of an economy in competitive equilibrium. There is a representative household and a representative firm. The household's utility functional is

$$U \equiv \int_0^\infty u(c_t) e^{-\rho t} dt,$$

with

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta},$$

where there is no population growth, and $\rho > 0$. The representative firm has a constant returns to scale per worker production function $f(k_t) = Ak_t^\alpha$. For simplicity, assume capital does not depreciate after production ($\delta = 0$). At every point in time, assume that the government institutes an income tax. That is, for every unit of income, the household must pay an amount τ to the government. The government then deposits the taxes in an offshore bank. Find the competitive equilibrium of this economy, using the following steps.

a) Write down representative household's maximization problem, solve it, and derive the 4 equations that characterize the solution. Explain in words, intuitively, what the Hamiltonian function means, and what the 4 equations represent.

b) Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form. Derive the 2 equations that characterize the solution.

c) What are the equilibrium conditions for this economy? Derive the government budget constraint.

d) Combine your answers to parts a) - c) and derive a pair of differential equations for the variables c and k.

e) Draw the phase diagram, carefully identifying (and deriving mathematically) all the important points.

f) Do the following comparative dynamics exercise: $\tau' > \tau = 0$. That is, compare the economy with and without the tax. As usual, the baseline economy starts in its steady state at time $t = 0$. The modified economy starts at time $t = 0$. Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of c and k for both cases. Carefully discuss your results. In particular, how does the tax affect the consumption/savings decision? Why?