
Part A: Answer any two of the three following questions.

A1. Growth and Technology Transfers

Suppose the world consists of two regions, the "North" and the "South." Total output and capital accumulation in region i (for $i = N,S$) are given by the following equations:

(1) $Y_i = K_i^{\alpha} [A_i (1 - c_{Li}) L_i]^{1-\alpha}$ where $0 < \alpha < 1$, $0 < c_{Li} < 1$

(2) $dK_i / dt = s_i Y_i$ where $0 < s_i < 1$

The variables are: Y_i = total output, A_i = technology, K_i = capital stock, L_i = labor force, and s_i = saving rate. Labor is assumed constant in both regions.

In the North, new technologies are developed by the R&D sector:

(3) $dA_N / dt = B\, c_{LN} L_N A_N$ where $B > 0$

In the South, technical improvements are made by learning from Northern technology:

(4) $dA_S / dt = \mu\, c_{LS} L_S (A_N - A_S)$ if $A_N > A_S$, otherwise $dA_S / dt = 0$; and $\mu > 0$

Here c_{LN} is the fraction of the Northern labor force employed in the R&D sector, and c_{LS} is the fraction of the Southern labor force learning Northern technology.

a. What is the long-run growth rate of output per worker in the North? Calculate and explain.

b. Define $Z = A_S / A_N$ as the South-North technology ratio. Find an expression for dZ / dt as a function of Z and the parameters of the model. Explain whether Z is stable. What is the long-run growth rate of output per worker in the South? Discuss.

c. Now assume $c_{LN} = c_{LS}$ and $s_N = s_S$. What is the ratio of output per worker in the South to output per worker in the North (y_S / y_N) when both regions have converged to their balanced growth paths? Calculate and explain.

A2. Wealth Effects

Economists have long argued that wealth effects can impact the effectiveness of macroeconomic policies. Let's address this issue by considering a closed economy with a wealth effect in the goods market; real wealth is held in the form of money and government bonds. The capital stock and technology are assumed fixed, and inflationary expectations are static. The complete aggregate supply/demand model is:

(1) $W/P = F_N(N,K)$	where $F_{NN} < 0$, $F_{NK} > 0$	(labor demand)
(2) $N = NS(W/P)$	where $NS_{W/P} > 0$	(labor market equilibrium)
(3) $Y = F(N,K)$	where F_N, $F_K > 0$	(production function)
(4) $Y = E(Y\text{-}T, R, V, G)$	where $0 < E_{Y-T} < 1$, $E_R < 0$, $0 < E_V < 1$, $E_G = 1$	(IS)
(5) $M/P = L(Y,R)$	where $L_Y > 0$, $L_R < 0$	(LM)

The variables are: W/P = real wage, N = labor, K = (fixed) capital stock, Y = real output, E = aggregate expenditures, T = taxes, R = (nominal) interest rate, G = government purchases, V = real wealth, B = government bonds, M = nominal money supply, P = price level, and L = real money demand.

Suppose the central bank conducts an open market purchase of government bonds. Analyze the effects of this policy for the following time horizons:

1. <u>Short Run (Keynesian world)</u>

   a. Show graphically and explain how/why the economy responds to the open market purchase in the short run. Discuss whether the presence of the wealth effect has any impact on the policy's effectiveness.

   b. Discuss how your answer changes if government bonds are not considered wealth.

2. <u>Long Run (Classical world)</u>

   a. Determine the long-run effects of the open market purchase by calculating and signing the relevant derivatives. Discuss whether the presence of the wealth effect has any impact on the policy's effectiveness. Be sure to consider the (super) neutrality of money.

   b. Now suppose that wealth also enters the labor supply function, i.e., $N = NS(W/P, V)$ with $NS_V < 0$. Show graphically and explain how/why the endogenous variables change in the long run under these circumstances.

A3. Statements

Select any four of the following statements and explain carefully why each is true, false, or uncertain. You must use graphical and/or mathematical analysis to support your arguments. Your score depends on the quality and completeness of your explanations.

1. In the Solow model with Hicks-neutral technology, technological progress affects neither the steady-state level of output per worker nor the economy's balanced growth path.

2. A full Fisher effect is possible only in the classical model.

3. The Policy Ineffectiveness Proposition always holds in the new classical model.

4. In a small open economy with fixed wages and prices, fiscal policy is more effective the higher the degree of capital mobility.

5. According to the Barro-Gordon model, the time-consistent equilibrium inflation rate will be lower the lower the natural rate of unemployment.

Part B: Answer <u>both</u> of the following two questions.

B1. Optimal Growth with a Revenue Requirement

Consider the optimal growth problem with technological growth. Assume the government needs to collect $\phi_t = \widehat{\phi} A_t$ units of revenue at each point in time [hint: the gov't takes $\widehat{\phi} A_t$ from the resource constraint]. Further, assume there is no population growth $(n = 0)$, and capital depreciates at the rate $\delta > 0$. Crusoe's utility functional is

$$\int_0^\infty \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt,$$

and the production function is $Y_t = F(K_t, A_t L_t)$, with the usual scale independence assumptions, so that $y_t = \frac{Y_t}{L_t}$. A_t grows at the constant rate $g > 0$.

a. Write down the optimal growth problem, the Hamiltonian function for this problem, and derive the four equations that characterize the solution.

b. Solve the first-order conditions to get a system of two ordinary differential equations in the variables (k, c) .Can you draw a phase diagram? Why/Why not?

c. Define $\widehat{k} = \frac{k}{A}, \widehat{y} = \frac{y}{A}$, and $\widehat{c} = \frac{c}{A}$. Derive a pair of differential equations for the variables $\widehat{c}$ and $\widehat{k}$.

d. Carefully draw the phase diagram, labelling the steady states on the graph and finding them algebraically.

e. Do the following comparative dynamics exercise: $\widehat{\phi}' > \widehat{\phi} = 0$. As usual, the baseline economy starts in its steady state at time $t = 0$. Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of c and k for both cases. Carefully discuss.

B2. Efficiency and Altruism in Overlapping Generations

Consider an economy consisting of an infinite sequence of two period lived, overlapping generations. Assume time t agents care about their offspring, and leave bequests b_{t+1}. N_t agents are born in period t, with $N_{t+1} = N_t$. In each period there is a single final good that is produced using a constant returns to scale technology with capital and labor as inputs. Let k_t denote the time t capital-labor ratio, and let $f(k_t)$ denote the intensive production function. Assume capital depreciates after production ($\delta > 0$). One unit of the final good that is not consumed at t converts into one unit of capital at $t + 1$. Let the utility of a generation born at time t be denoted by V_t, where

$$V_t = u(c_{1,t}, c_{2,t+1}) + \frac{1}{1+\rho} V_{t+1}$$

Let $\rho > 0$ be the discount factor, and

$$u(c_{1,t}, c_{2,t+1}) = \ln(c_{1,t}) + \frac{1}{1+\rho} \ln(c_{2,t+1})$$

a. Solve V_t recursively forward. What does your solution look like?

b. Write down the household's maximization problem and the first-order conditions.

c. Write down the firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form. Derive the 2 equations that characterize the solution.

d. What are the three equilibrium conditions for this economy? Explain them.

e. Now assume the economy is at the non-trivial steady-state. Combine your results in b.-d. to get to an equation that defines the marginal product of capital. Carefully discuss. Is this Pareto optimal? Why/Why not?