Ph.D. Core Exam -- Macroeconomics **11 August 2014 -- 8:00 am to 3:00 pm**

Part A: Answer question A1 (required), plus <u>either</u> question A2 <u>or</u> A3.

A1 (required): Cutting Taxes

Conventional wisdom holds that tax cuts stimulate the economy. As professional economists, we often have the task of pointing out that "it depends" on a number of conditions, including the degree of wage and price flexibility or the degree of openness, among others. Let's clarify matters by analyzing the effects of a debt-financed tax cut under the following scenarios.

1. <u>*Closed Economy: Short vs. Long Run*</u>

Assume the economy's capital stock and technology are fixed, and inflationary expectations are static. The aggregate demand side of the economy is described by:

(IS) $Y = E(Y\text{-}T,R,G)$ where $0 < E_{Y-T} < 1$, $E_R < 0$, $E_G = 1$

(LM) $M/P = L(Y,R)$ where $L_Y > 0$, $L_R < 0$

The variables are: Y = real output/income, E = aggregate expenditures, T = taxes, $R = r$ = nominal/real interest rate, G = government purchases of goods and services, M = nominal money supply, P = price level, and L = real money demand.

a. <u>For the short run, when wages and prices are fixed</u>:
   i. Show graphically and explain how/why the economy responds to the debt-financed tax cut.
   ii. Explain how your answer changes if money demand depends on disposable real income $(Y\text{-}T)$ rather than total real income (Y).

b. <u>For the long run, when wages and prices are flexible</u>:
   i. Explain how/why the economy responds to the debt-financed tax cut and calculate/sign the relevant derivatives.
   ii. Explain how your answer changes if there is a real wealth effect in the goods market.

2. <u>*Small Open Economy: Fixed vs. Flexible Exchange Rates*</u>

Assume perfect capital mobility. The economy is described by a standard Mundell-Fleming model with fixed wages/prices, static inflationary expectations, and static exchange rate expectations.

Using graphical and/or mathematical analysis, explain how/why the economy responds to the debt-financed tax cut under fixed versus flexible exchange rates.

A2: Growth and Education

Consider the following Solow economy, where total output *(Y)* is a constant-returns-to-scale production function of physical capital *(K)* and effective human capital *(AH)*. Human capital is formed through education *(E)*.

(1)	$Y = K^{\alpha} (AH)^{1-\alpha}$	where $0 < \alpha < 1$	(production function)
(2)	$H = L\, e^{\theta E}$	where $\theta > 0$	(human capital formation)
(3)	$dK/dt = sY$	where $0 < s < 1$	(physical capital accumulation)
(4)	$dL/dt = nL$	where $n > 0$	(labor accumulation)
(5)	$dA/dt = gA$	where $g > 0$	(technical progress)

The remaining variables are: A = human-capital-augmenting technology, L = raw labor, α = income share of physical capital, θ = return to education, s = saving rate, n = population growth rate, g = rate of technical progress.

a. Characterize the steady-state equilibrium for this economy by (i) showing the steady-state equilibrium in a Solow graph, (ii) calculating output per worker $(y = Y/L)$ in steady-state, and (iii) deriving the growth rate of output per worker on the balanced growth path.

b. Now suppose the government permanently increases the required years of schooling or vocational training. (i) Illustrate graphically how the economy is affected by this ceteris paribus change. (ii) Explain how/why both the level and the growth rate of output per worker respond over time to the increase in educational requirements.

c. Discuss how this model can help explain the current world income distribution and/or cross-country differences in growth rates.

A3: Statements

Select any three of the following statements and explain carefully why each is true, false, or uncertain in all its parts. You must use graphical and/or mathematical analysis to support your arguments. Your score depends on the quality and completeness of your explanations.

1. According to the human-capital version of the Solow model, an increase in the country's skill ratio $(h = H/L)$ will have permanent level and growth rate effects on output per worker.

2. The neutrality of money depends on the absence or presence of a wealth effect in the goods market of the economy.

3. In a small open economy operating under fixed exchange rates and zero capital mobility, policymakers have no way of stimulating the domestic economy.

4. According to the stochastic IS-LM model (i.e., the Poole model), policymakers can either fix the interest rate or the money supply to stabilize output in the face of random shocks that hit the economy.

Part B: Answer B1 (required), plus either question B2 or B3.

B1 (required): Taxes and Dynamic Inefficiency in an OLG Model

Consider an economy consisting of an infinite sequence of two period lived, overlapping generations. N_t agents are born in period t, with $N_{t+1} = (1+n)N_t$. In each period there is a single final good that is produced using a constant returns to scale technology with capital and labor as inputs. Let k_t denote the time t capital-labor ratio, and let $f(k_t)$ denote the intensive production function. Let f have the Cobb-Douglas form $f(k_t) = Ak_t^{\alpha}$, with $0 < \alpha < 1$. One unit of the final good that is not consumed at t converts into one unit of capital at $t+1$. Capital depreciates after production, with $\delta \in (0,1)$. Agents have the utility function

$$u(c_{1,t}, c_{2,t+1}) = \frac{c_{1,t}^{1-\theta} - 1}{1-\theta} + (1+\rho)^{-1} \frac{c_{2,t+1}^{1-\theta} - 1}{1-\theta}$$

with $\theta \to 1$.

For each unit of assets a_t owned by agents in their second period of life, the government gives them a subsidy of σ_{t+1}. In order to balance the budget, the government imposes a tax τ_t on labor income w_t.

a) Write down the household's maximization problem and derive the equations that characterize the solution $(a_t, c_{1,t}, c_{2,t+1})$. Discuss. Do taxes and subsidies appear in these equations? Why or why not?

b) Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form.

c) What are the equilibrium conditions for this economy? What is the government budget constraint?

d) Combine your answers to parts (a) - (c) and derive a *Law of Motion* (*LoM*) equation that defines a difference equation for the variable k. Get rid of all prices. Looking at it, can we say anything about a steady-state solution? Can you graph the *LoM?*

e) Is the non-trivial steady-state in the Competitive Equilibrium (CE) Pareto Optimal (PO)? Carefully show and explain why, or why not. Under what conditions will the CE be PO? Can you find an optimal tax, so that the CE is PO?

f) Do the following comparative dynamics exercise. Initially, the CE economy is with $\sigma = \tau = 0$, and now the government imposes the optimal tax and subsidy rates that you found in part e). Draw (i) LoM for both cases, indicating what is different, and (ii) the time paths of the logs of c and k for both cases.

B2: Sales Taxes in an Optimal Growth Model

Consider the Ramsey model of an economy in competitive equilibrium. There is a representative household and a representative firm. The household's utility functional is

$$U \equiv \int_0^\infty u(c_t)e^{-\rho t}dt,$$

with

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta},$$

where there is no population growth, and $\rho > 0$.The representative firm has a constant returns to scale per worker production function $f(k_t) = Ak_t^\alpha$. For simplicity, assume capital does not depreciate after production ($\delta = 0$). At every point in time, assume that the government institutes a consumption tax (aka sales tax). That is, for every unit of consumption that it chooses, the household must pay an amount τ to the government. The government then deposits the taxes in an offshore bank. Find the competitive equilibrium of this economy, using the following steps.

a) Write down representative household's maximization problem, solve it, and derive the 4 equations that characterize the solution. Explain in words, intuitively, what the Hamiltonian function means, and what the 4 equations represent. Does τ show up here? Explain why or why not?

b) Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form. Derive the 2 equations that characterize the solution. Does τ show up here? Explain why or why not?

c) What are the equilibrium conditions for this economy? Does τ show up here? Explain why or why not? Derive the government budget constraint.

d) Combine your answers to parts a) - c) and derive a pair of differential equations for the variables c and k.

e) Draw the phase diagram, carefully identifying (and deriving mathematically) all the important points.

f) Do the following comparative dynamics exercise: $\tau' > \tau = 0$. That is, compare the economy with and without a sales tax. As usual, the baseline economy starts in its steady state at time $t = 0$. The modified economy starts at time $t = 0$. Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of c and k for both cases. Carefully discuss your results. In particular, how does the tax affect the consumption/savings decision? Why?

B3: Optimal Growth with Capital Externalities

Consider the AK model of an economy in competitive equilibrium, where there are capital externalities. There is a representative household and a representative firm. The household's utility functional is

$$U \equiv \int_0^\infty u(c_t)e^{-\rho t}dt,$$

with

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta},$$

where $1 > \rho > n = 0$, and $\theta > 0$.

The representative firm has a production function $F\left[K_t, \bar{K}_t, L_t\right] = K_t^\alpha(\bar{K}_t L_t)^{1-\alpha}$, where $\bar{K}$ is the total quantity of capital in the economy, normalize $L = 1$, and assume capital does not depreciate after production ($\delta = 0$). Find the competitive equilibrium of this economy, using the following steps.

a) Write down representative household's maximization problem, solve it, and derive the 4 equations that characterize the solution.

b) Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form. Derive the 2 equations that characterize the solution.

c) What are the 4 equilibrium conditions for this economy?

d) Combine your answers to parts a) - c) and derive a pair of differential equations for the variables c and k. Can you draw a phase diagram? If yes, draw the phase diagram, carefully identifying (and deriving mathematically) all the important points. Is there a balanced growth path? Show it on the graph, and derive it's slope.

e) What is the growth rate of the economy? What about transitional dynamics?

f) Do the following comparative dynamics exercise: $\theta' > \theta$. As usual, the baseline economy starts in its balanced growth path at time $t = 0$. The modified economy starts at time $t = 0$. Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of the logs of c and k for both cases.