Ph.D. Core Exam -- Macroeconomics **5 January 2015 -- 8:00 am to 3:00 pm**

Part A: Answer question A1 (required), plus <u>either</u> question A2 <u>or</u> A3.

A1 (required): Ending Quantitative Easing

Now that the U.S. economy has fully recovered from the "Great Recession" of 2008-2009, the Fed (i.e., the U.S. central bank) has announced an end to its policy of quantitative easing (aka "QE"). Over the next year or so, the Fed plans to sell off most of the Treasury notes and other securities it bought under successive "QE" rounds. Your task is to analyze the effects of this open-market sale of government bonds and other securities under the following scenarios.

1. *<u>Domestic Effects on the U.S. Economy: Short vs. Long Run</u>*

Assume the U.S. economy is a closed economy with a fixed capital stock and given technology. Inflationary expectations are static. The aggregate demand side of the economy is described by:

(IS) $Y = E(Y-T,R,V,G)$ where $0 < E_{Y-T} < 1$, $E_R < 0$, $0 < E_V < 1$, $E_G = 1$

(LM) $M/P = L(Y,R)$ where $L_Y > 0$, $L_R < 0$

The variables are: Y = real output/income, E = aggregate expenditures, T = taxes, R = (nominal/real) interest rate, $V = (M+B)/P$ = real wealth, G = government purchases, M = nominal money supply, B = government bonds held by general public, P = price level, L = real money demand.

a. <u>For the short run, when wages and prices are fixed</u>:
   i. Show graphically and explain how/why the U.S. economy responds to the open market sale.
   ii. Explain how your answer changes if investment is highly interest-elastic.

b. <u>For the long run, when wages and prices are flexible</u>:
   i. Explain how/why the U.S. economy responds to the open market sale and calculate/sign the relevant derivatives.
   ii. Explain how your answer changes if government bonds are not considered wealth.

2. *<u>Effects on U.S. Trading Partners: Fixed vs. Flexible Exchange Rates</u>*

The end of "QE" will affect world interest rates in the same direction as U.S. interest rates. Consider the case of a small open economy that trades with the U.S. The country's economy is described by a standard Mundell-Fleming model with fixed wages/prices, static inflationary expectations, and static exchange rate expectations.

Assuming perfect capital mobility, use graphical and/or mathematical analysis to explain how/why the small open economy responds to the Fed-induced change in world interest rates under fixed versus flexible exchange rates.

A2: Growth and Technology Transfers

Suppose the world consists of two regions, the "North" and the "South." Total output and capital accumulation in region i (for $i = N,S$) are given by the following equations:

(1) $Y_i = K_i^{\alpha} [A_i (1 - c_{Li}) L_i]^{1-\alpha}$ where $0 < \alpha < 1$, $0 < c_{Li} < 1$

(2) $dK_i / dt = s_i Y_i$ where $0 < s_i < 1$

The variables are: Y_i = total output, A_i = technology, K_i = capital stock, L_i = labor force, and s_i = saving rate. Labor is assumed constant in both regions.

In the North, new technologies are developed by the R&D sector:

(3) $dA_N / dt = B\, c_{LN} L_N A_N$ where $B > 0$

In the South, technical improvements are made by learning from Northern technology:

(4) $dA_S / dt = \mu\, c_{LS} L_S (A_N - A_S)$ if $A_N > A_S$, otherwise $dA_S / dt = 0$; and $\mu > 0$

Here c_{LN} is the fraction of the Northern labor force employed in the R&D sector, and c_{LS} is the fraction of the Southern labor force learning Northern technology.

a. What is the long-run growth rate of output per worker in the North? Calculate and explain.

b. Define $Z = A_S / A_N$ as the South-North technology ratio. Find an expression for dZ / dt as a function of Z and the parameters of the model. Explain whether Z is stable. What is the long-run growth rate of output per worker in the South? Discuss.

c. Now assume $c_{LN} = c_{LS}$ and $s_N = s_S$. What is the ratio of output per worker in the South to output per worker in the North (y_S / y_N) when both regions have converged to their balanced growth paths? Calculate and explain.

A3: Statements

Select <u>any three</u> of the following statements and explain carefully why each is true, false, or uncertain in all its parts. You must use graphical and/or mathematical analysis to support your arguments. Your score depends on the quality and completeness of your explanations.

1. Higher population growth causes the level of steady-state output per worker to fall according to the Solow growth model, and the steady-state growth rate of output per worker to rise according to the R&D growth model.

2. Whether the Fisher effect is full or partial depends solely on the interest elasticity of real money demand.

3. Given rational expectations, pre-announced policies do not affect output in the short run whereas surprise policies may end up destabilizing the economy.

4. According to the Barro-Gordon model, the time-consistent equilibrium inflation rate will be lower the lower the natural rate of unemployment and the steeper the short-run Phillips curve.

Part B: Answer B1 (required), plus either question B2 or B3.

B1 (required): Sales Taxes in an Optimal Growth Model

Consider the Ramsey model of an economy in competitive equilibrium. There is a representative household and a representative firm. The household's utility functional is

$$U \equiv \int_0^\infty u(c_t)e^{-\rho t}dt,$$

with

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta},$$

where there is no population growth, and $\rho > 0$. The representative firm has a constant returns to scale per worker production function $f(k_t) = Ak_t^\alpha$. For simplicity, assume capital does not depreciate after production ($\delta = 0$). At every point in time, assume that the government institutes a consumption tax (aka sales tax). That is, for every unit of consumption that it chooses, the household must pay an amount τ to the government. The government then deposits the taxes in an offshore bank. Find the competitive equilibrium of this economy, using the following steps.

a) Write down the representative household's maximization problem, solve it, and derive the 4 equations that characterize the solution. Explain in words, intuitively, what the Hamiltonian function means, and what the 4 equations represent. Does τ show up here? Explain why or why not?

b) Write down the firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form. Derive the 2 equations that characterize the solution. Does τ show up here? Explain why or why not?

c) What are the equilibrium conditions for this economy? Does τ show up here? Explain why or why not? Derive the government budget constraint.

d) Combine your answers to parts a) - c) and derive a pair of differential equations for the variables c and k.

e) Draw the phase diagram, carefully identifying (and deriving mathematically) all the important points.

f) Do the following comparative dynamics exercise: $\tau' > \tau = 0$. That is, compare the economy with and without a sales tax. As usual, the baseline economy starts in its steady state at time $t = 0$. The modified economy starts at time $t = 0$. Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of c and k for both cases. Carefully discuss your results. In particular, how does the tax affect the consumption/savings decision? Why?

B2: Efficiency and Altruism in Overlapping Generations

Consider an economy consisting of an infinite sequence of two period lived, overlapping generations. Assume time t agents care about their offspring, and leave bequests b_{t+1}. N_t agents are born in period t, with $N_{t+1} = N_t$. In each period there is a single final good that is produced using a constant returns to scale technology with capital and labor as inputs. Let k_t denote the time t capital-labor ratio, and let $f(k_t)$ denote the intensive production function. Assume capital depreciates after production ($\delta > 0$). One unit of the final good that is not consumed at t converts into one unit of capital at $t+1$. Let the utility of a generation born at time t be denoted by V_t,where

$$V_t = u(c_{1,t}, c_{2,t+1}) + \frac{1}{1+\rho} V_{t+1}$$

Let $\rho > 0$ be the discount factor, and

$$u(c_{1,t}, c_{2,t+1}) = \ln(c_{1,t}) + \frac{1}{1+\rho} \ln(c_{2,t+1})$$

a. Solve V_t recursively forward. What does your solution look like?

b. Write down the household's maximization problem and the first-order conditions.

c. Write down the firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form. Derive the 2 equations that characterize the solution.

d. What are the three equilibrium conditions for this economy? Explain them.

e. Now assume the economy is at the non-trivial steady-state. Combine your results in b.-d. to get to an equation that defines the marginal product of capital. Carefully discuss. Is this Pareto optimal? Why/Why not?

B3: Optimal Growth with a Revenue Requirement

Consider the optimal growth problem with technological growth. Assume the government needs to collect $\phi_t = \widehat{\phi} A_t$ units of revenue at each point in time [hint: the government takes $\widehat{\phi} A_t$ from the resource constraint]. Further, assume there is no population growth $(n = 0)$, and capital depreciates at the rate $\delta > 0$. Crusoe's utility functional is

$$\int_0^\infty \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt,$$

and the production function is $Y_t = F(K_t, A_t L_t)$, with the usual scale independence assumptions, so that $y_t = \frac{Y_t}{L_t}$. A_t grows at the constant rate $g > 0$.

a. Write down the optimal growth problem, the Hamiltonian function for this problem, and derive the four equations that characterize the solution.

b. Solve the first-order conditions to get a system of two ordinary differential equations in the variables (k, c). Can you draw a phase diagram? Why/Why not?

c. Define $\widehat{k} = \frac{k}{A}, \widehat{y} = \frac{y}{A}$, and $\widehat{c} = \frac{c}{A}$. Derive a pair of differential equations for the variables $\widehat{c}$ and $\widehat{k}$.

d. Carefully draw the phase diagram, labelling the steady states on the graph and finding them algebraically.

e. Do the following comparative dynamics exercise: $\widehat{\phi}' > \widehat{\phi} = 0$. As usual, the baseline economy starts in its steady state at time $t = 0$. Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of c and k for both cases. Carefully discuss.