**PhD/MA. Econometrics Examination**
**January 13., 2010**

**Total Time: 8 hours**
**MA students are required to answer from A and B.**
**PhD students are required to answer from A, B, and C.**

## PART A
## (ANSWER ANY TWO)

**Question 1.**

a. Is $f(y) = (1/2)y(1-y) \quad 0 \le y \le 2$ a probability density function?

b. The cumulative distribution function for random variable $Y$ is given by $F(y) = y/5 \quad 0 \le y \le 5$. Find the probability density function for $Y$.

c. If $P(B) = .05$, $P(A \mid B) = .80$, and $P(A \mid \overline{B}) = .50$ find $P(B \mid A)$.

**Question 2.**

a. $U$ is a random variable with probability density function $f(u) = 2e^{-2u} \quad 0 < u < \infty$. Let $g(U) = e^{-U}$, what is $E[g(U)]$?

b. If $E(Y_1) = 2$, $E(Y_2) = 1$, $V(Y_1) = \frac{1}{2}$, $V(Y_2) = 1$, and $Cov(Y_1, Y_2) = -\frac{1}{2}$ find the mean and variance of the random variable $U = 2Y_1 - Y_2$.

c. Is it possible that the two random variables $Y_1$ and $Y_2$, have standard deviations $\sigma_{Y_1} = 2.5$, $\sigma_{Y_2} = -.9$, respectively, $\text{cov}(Y_1, Y_2) = 2.7$ and correlation $\rho = -1.2$? Explain.

**Question 3.**

a. Find the maximum likelihood estimator of $\beta$ where $Y_i$ is independently and identically distributed with probability density function (the exponential density):

$$f(y) = \frac{1}{\beta} e^{-y/\beta} \quad y > 0.$$

b. Let $\hat{u}_i$ be the OLS residuals for the model $y_i = \beta + u_i$. Given that $\frac{1}{\sigma^2} \sum_{i=1}^{T} \hat{u}_i^2 \sim \chi^2(n-1)$ and $\tilde{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{T} \hat{u}_i^2$ show that $E(\tilde{\sigma}^2) \ne \sigma^2$, i.e. $\tilde{\sigma}^2$ is biased. Use the result to create an unbiased estimator of $\sigma^2$ denoted by $\hat{\sigma}^2$.

c. Normality is often assumed in applied work. State the *central limit theorem* and discuss how this result is useful.

**PART B**
**(ANSWER ANY TWO)**

**Question 4.**
A. Consider the following stock-adjustment model:

$$\ln Y(t)^* = a_0 + a_1 \ln X(t) + U(t) \qquad (1)$$

where Y(t)* is desired inventory of cars in billions of $; X(t) is the oil price ($ per gallon)

The stock-adjustment mechanism is as follows:

$$\frac{Y(t)}{Y(t-1)} = \left(\frac{Y(t)^*}{Y(t-1)}\right)^d \quad 0 < d < 1 \qquad (2)$$

(Hint: take the log of the equation (2) to make it operational.)

Combine equation (1) and (2) to come up with a Koyck type distributed lag model and discuss the estimation method. What is the interpretation of *d*?

B. Given the following distributed lag type model for the annual car inventory data (1980 – 2004):

$$\ln Y(t) = \beta_0 + \beta_1 \ln Y(t-1) + \beta_2 \ln X(t) + V(t)$$

Where Y(t) = Inventory of cars in billions of $; X(t) = oil price ($ per gallon).
The following estimates were found: $\hat{\beta}_0 = 1.34, \hat{\beta}_1 = .62, \hat{\beta}_2 = .42$.

a. Calculate the long-run multiplier effect of oil price shock on car inventory.
b. Calculate the half life value and interpret the result.
c. Calculate the impact multipliers at lag 0, 1, and 2.

**Question 5.**
Given the following simple slope-only model: $y(t) = \beta x(t) + u(t)$
where $u(t)$ is heteroskedastic, i.e., $\sigma^2$(t) = exp($g_0 z(t)$), exp = exponent, and $g_0$ is a scalar parameter.

a. Show that the OLS estimator of $\beta$ is still unbiased.
b. Derive the variance of $\hat{\beta}$.
c. Assuming the above heteroskedastic form, present the Var-Cov(U) matrix, and present its transformation matrix, P.
d. Present the weighted GLS iterative method. Show all the iterative steps.
e. What is difference between GLS versus OLS?

**Question 6.**
Consider the following linear model in matrix notation

$$\mathbf{Y} = \mathbf{X}\beta + \mathbf{U}$$

a. Use the method of your choice --OLS or MLE—to derive the estimator of $\beta$.
b. Now derive the Var-Cov(**U**)
c. Demonstrate the Gauss-Markov theorem that $\hat{\beta}_{OLS}$ is BLUE.
d. Derive the Var-Cov matrix of the OLS estimator of $\beta$ (either use OLS method or MLE-hessian matrix method)

## PART C
## (ANSWER TWO QUESTIONS)

**Question 7.**
Calculate the autocorrelation functions up to 3 lags for the following model:

$$y_t = \varphi_1 y_{t-1} + e_t$$

b. Plot the autocorrelation functions for the following values, each separately
   i. $\varphi_1 = .7$ ii. $\varphi_1 = -.8$

c. What is the difference between the unit root versus the long-memory models.
d. Explain the meaning of cointegration with examples.
e. Explain the meaning of the spurious regression phenomenon with examples.

**Question 8.**
Given the following nonlinear consumption function:

$$y(t) = \left( \frac{\alpha}{1 + \exp(\beta x(t))} \right) + u(t)$$

Where $\alpha$, and $\beta$ are unknown parameters.

a. You may express this function generically: $y(t) = f(\alpha, \beta, x(t)) + u(t)$ and show how you might estimate using the method of non-linear least squares OR the Newton-Raphson (NR) method of estimating it using a distribution of your choice.
b. If you were to test the unbiasedness of the estimators (of $\alpha$ and $\beta$) of the two methods – non-linear least squares versus the NR method—how would you set up a Monte Carlo experiment. (Show all the steps in details.)

**Question 9.**
Consider the following structural model

$$y_1(t) = \alpha_0 + \alpha_1 x_1(t) + \alpha_2 y_2(t) + \alpha_3 y_1(t-1) + u_1(t)$$
$$y_2(t) = \beta_0 + \beta_1 x_1(t) + \beta_2 x_2(t) + u_2(t)$$

a. List the endogenous variables. List the predetermined variables.
b. Write the above model in a general matrix notation:

$$\mathbf{Y}\Gamma + \mathbf{X}\Delta + \mathbf{U} = \mathbf{0}$$

c. Using the general method, test the identification condition for these equations – one equation at a time.
d. Derive the Var-Cov(**U**).
e. Present the 3SLS method of estimation.
f. Consider a generic equation as follows to represent the first equation of the above model.

$$\mathbf{y}_1 = \mathbf{Y}_1\gamma_1 + \mathbf{X}_1\delta_1 + \mathbf{u}_1$$

Now, show that the 2SLS estimator of this equation is consistent.

$$\mathbf{y}_1 = \mathbf{Z}_1\beta_1 + \mathbf{u}_1$$