Ph.D. MICROECONOMICS CORE EXAM
August 2015

This exam is designed to test your broad knowledge of microeconomics. There are three sections: one required and two choice sections. You must complete both problems in the required section and one choice problem in each of the two choice sections, giving you a total of four problems to complete during the allotted time. The required problems are in section A and the choice problems are in sections B and C. If you should answer more than one choice question in a section, only the first will be considered.

IMPORTANT. You are expected to adhere to the following guidelines in completing the exam for your answer to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number all pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number.**
- Provide clear, concise discussion to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Explain your steps in any mathematical derivations. Simplify your final answers completely.
- When you turn in your exam answers double check to make sure you have included all the pages to each question number, in order. The pages you submit as your answer are the only ones that will be considered.
- To simplify copying, please leave 1 inch borders.

PART A: REQUIRED QUESTIONS

Both problems in Part A (A1 and A2) are required. Answer all parts of all questions.

QUESTION A1

Albuquerque has recently benefited from the construction of new roads (e.g. Paseo Del Norte interchange on I-25). New roads reduce traffic jams and vehicular congestion that had earlier existed. Additionally, new roads increase the efficiency with which vehicles can travel; all vehicles have an ideal speed at which their engines run at the highest efficiency. However, what is commonly overlooked in urban planning is that the construction of new roads also reduces air pollution as more cars are able to drive efficiently.

Develop a simple GE model of the Albuquerque economy to demonstrate the consequences of reduced air pollution throughout the city and how market based mechanisms could be employed to achieve a Pareto efficient outcome.

To help with your model development, assume the economy consists of 2 sectors, transportation improvement projects (t) and health (h) with prices p_i, $i = t, h$. Let all consumers in the economy be identical and thus represented by a single consumer who derives utility from consuming both of the produced goods, $U(t, h)$. Further, consider only a single factor of production, labor, that the consumer is endowed with in the amount $\bar{L}$. The consumer receives no utility from consumption of labor and sells all of it to producing sectors at wage rate w, which you can assume is the numeraire good. Production in all sectors is characterized by quasi-concave production functions, each a function of labor employed in the sector, l_i, $i = t, h$. The health sector production function is also a function of air quality, which is affected by the transportation improvement projects. Specifically, the production of transportation and health are as follows, $t = T(l_T)$ and $h = H(l_h, D(t))$, where $D(t)$ represents the pollution discharges and $\frac{\partial H}{\partial D} < 0$ and $\frac{\partial D}{\partial t} < 0$.

a. Derive and interpret the socially (Pareto) optimal solution to the problem. Assume interior solutions only.
b. Derive and interpret the competitive solution to the problem. Assume interior solutions only.
c. Explain the difference between part a. and b. Relate your findings to what you know about the first fundamental theorem of welfare economics. A graph could be useful here.
d. Derive and interpret the optimal tax/subsidy needed to restore the Pareto solution in a competitive world.

QUESTION A2

Suppose you have a consumer with the following utility function:

$$u(x, y) = 3x + y$$

a. Derive the indirect utility.
b. Derive the expenditure function.
c. $P_x = \$2;\ P_y = \$1; m = \$20.$ Suppose the government imposes a sales tax *t* = \$2 on good *x*. How much would the consumer have to be compensated to be as well off as they were before the tax was imposed? What if $P_y = \$2$, would your answer change?
d. Suppose the government instead imposes an income tax. Is the consumer better off with the sales tax or the income tax? Is this result consistent with WARP? Explain.

PART B: CHOICE QUESTIONS

Answer all parts of either question B1 or B2. If you complete more than one problem, only B1 will be considered.

QUESTION B1

a. Prove whether the following statement is true or false: Hicksian demand $\mathbf{x}^h(\mathbf{p},u)$ is homogeneous of degree one in utility.
b. Explain your above result intuitively.
c. Suppose an individual's preferences are represented by $U(x_1, x_2, x_3, x_4)=min\{ x_1+ x_2, x_3\}+ x_4$. Assume prices and income are strictly positive. Find the Marshallian demand. (Note: you may ignore all cases of equal prices.)
d. Provide an example of a set of goods (x_1 - x_4) that could be described by this type of utility function.

QUESTION B2

The perfectly competitive Dig-It School offers instruction in driving backhoes (see Figure 1 below). The School's production function is $q = 10 * \min\{k, l\}^{\gamma}$, where q is the number of students trained, k is the number of backhoes, l is the number of instructors, and γ is a parameter indicating returns to scale in the production function.

a. What restrictions are required on the value of γ in order for a profit-maximizing solution to exist and why? (Assume $q > 0$.)
b. Assume $\gamma=0.5$. Calculate the School's total cost function and profit function. (Use p to denote the price paid by each student.)
c. If the cost of a backhoe is \$1000 per student, the cost of an instructor is \$500 per student, and the price paid by students is \$600, how many students will Dig-It train and what are its profits?
d. If the students' willingness to pay rises to \$900, how much will profits change?

Figure 1: Picture of a backhoe

PART C: CHOICE QUESTIONS

Answer all parts of either question C1 or C2. If you complete more than one problem, only C1 will be considered.

QUESTION C1

The unruly James and Dean are playing their more dangerous variant of chicken (again). They've noticed that their payoff for being perceived as "tough" varies with the size of the crowd. The larger the crowd on hand, the more glory and praise each receives from drive straight when his opponent swerves. Smaller crowds, of course, have the opposite effect. Let $k > 0$ be the payoff for appearing "tough." The game may now be represented as:

		Dean	
		Swerve	Straight
James	Swerve	0,0	-1,k
	Straight	k,-1	-10,-10

a. Expressed in terms of k, with what probability does each driver play Swerve in the mixed-strategy Nash equilibrium? Do James and Dean play Swerve more or less often as k increases? Explain.
b. In terms of k; what is the expected value of the game to each player when both are playing the mixed-strategy Nash equilibrium found in part a.? Explain.
c. At what value of k do both James and Dean mix 50-50 in the mixed-strategy equilibrium? Explain.
d. James and Dean decide to play the chicken game repeatedly (say, in front of different crowds of reckless youths). Moreover, because they don't want to collide, they collude. They alternate between the two pure-strategy equilibrium, so that half the time they play (Swerve, Straight) and half the time they play (Straight, Swerve). Assuming they play an even number of games, what is the average payoff per set of games when they alternate between the two pure-strategy equilibria? How large must k be for the average payoff to be positive under the alternating scheme.

QUESTION C2

Crude oil is transported across the globe in enormous tanker ships called Very Large Crude Carriers (VLCC's). By 2001, more than 92% of all new VLCC's were built in South Korea and Japan. Assume that the price of new VLCC's (in millions of dollars) is determined by the function $P = 180 - Q$, where $Q = q_{Korea} + q_{Japan}$. (That is, assume that only Japan and Korea produce VLCC's, so they are a duopoly). Assume that the cost of building each ship is $30 million in both Korea and Japan. That is, $c_{Korea} = c_{Japan} = 30$, where the per-ship cost is measured in millions of dollars.

a. Write the profit function for each country in terms of q_{Korea} and q_{Japan} and either c_{Korea} or c_{Japan}. Find each country's best-response function.
b. Using the best-response function found in part a, solve for the Nash equilibrium quantity of VLCC's produced by each country per year. What is the price of a VLCC? How much profit is made in each country?
c. Labor costs in Korean shipyards are actually much lower than in their Japanese counterparts. Assume now that the cost per ship in Japan is $40 million and that in Korea it is only $20 million. Given $c_{Korea} = 20$ and $c_{Japan} = 40$, what is the market share of each country? What are the profits for each country? Discuss and compare your answer to part b.
d. Suppose China decides to enter the VLCC construction market. The duopoly now becomes a triopoly, so that although price is still $P = 180 - Q$, quantity is now given by $Q = q_{Korea} + q_{Japan} + q_{China}$. Assume that all three countries have a per-ship cost of $30 million: $c_{Korea} = c_{Japan} = q_{China} = 30$. With this new information, write the profit functions for each country. Find each country's best response rule and interpret. Find the profit and quantity produced in each country. Be sure to explain what happens to the price of a VLCC in the new triopoly relative to the duopoly situation in part a and b.