This exam is designed to test your broad knowledge of microeconomics. There are three sections: one required and two choice sections. You must complete both problems in the required section (section A) and one choice problem in each of the two choice sections (sections B & C), giving you a total of four problems to complete during the allotted time. If you should answer more than one choice question in a section, only the first will be considered.

IMPORTANT. You are expected to adhere to the following guidelines in completing the exam for your answer to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number all pages with the question number and the page number (e.g., A1-1, A1-2, etc.) and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number.**
- Provide clear, concise discussion to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- You must show all of your steps; you must annotate your work (that is, explain your procedure and what you're doing at each step); and in all cases you must provide a brief explanation of your answer.
- Simplify your final answers completely.
- When you turn in your exam answers double check to make sure you have included all the pages to each question number, in order. The pages you submit as your answer are the only ones that will be considered.
- To simplify copying, please leave one-inch borders on top, bottom and sides.

PART A: Required Questions - Answer both of the following questions

<u>QUESTION A1: "OPTIMALITY"</u>

Consider the producer with the indirect cost function of

$$\tilde{C}(w,r,q) = \frac{q\left(w^{0.5} + r^{0.5}\right)^2}{10},$$

where *w* is the price of labor, *L, r* is the price of capital, *K*, and *q* is the quantity produced.

a) Find the conditional input demands (Hicksian) for this producer and explain what will determine if more labor or capital is used in an optimal outcome.
b) Suppose the producer must produce exactly 50 units of the good to fulfill a contract obligation. The price of the good is \$100, *w = \$40* per unit, and *r = \$50* per unit. How much labor and capital will the producer optimally use? Explain your answers.
c) Find a production function that is consistent with the indirect cost function.
d) Consider the same input and output prices as in b). Find the optimal output level for the firm. Explain your answer.

<u>QUESTION A2: "PRICES"</u>

Consider the case where there are two firms in a market. The firms have slightly differentiated products and so are imperfect substitutes for each other. Let the demand for the goods offered by the firms be described by the following:

$q_1 = a - b_1 p_1 + b_2 p_2$ and $q_2 = a - b_1 p_2 + b_2 p_1$ where $b_1 > b_2$. Each firm has cost function of $TC_i = cq_i \ (i = 1,2)$.

In all cases, show your work and annotate your steps and answers.

a) Explain the significance of the restriction $b_1 > b_2$.
b) Find the Bertrand pricing solution for these two firms if they move simultaneously.
c) Given your answer in b), what has to hold for profits to be positive for these firms?
d) Suppose firm 1 moves first. For simplicity, let *a=10,* b_1 *= 2,* b_2 *=1, and c=2.* Find the outcome for this game and explain.

PART B: Choice. Answer either B1 or B2. If you answer both, only B1 will be graded.

<u>QUESTION B1: "I'M JUST NOT SURE"</u>

Decisions are often made under uncertainty. Answer each of the following.

a) Suppose an individual has a von Neumann-Morgenstern utility function, $u(w)$, where *w* is wealth. If *u* is linear, the individual is considered risk neutral and if *u* is concave we say the individual is risk averse. Let there be a simple gamble between two alternatives: w_1 and w_2 (where $w_1 > w_2$) with probabilities ρ_1 and ρ_2 (where $\rho_1 + \rho_2 = 1$). Show that if a risk averse individual prefers the gamble $(w_1, w_2, \rho_1, \rho_2)$ to the certain wealth position, w_0, then so does the risk neutral individual.

b) Three friends (Stu, David and Mike) go to the racetrack together. David prefers to avoid all risks. Stu and Mike are less risk averse. David will not bet on a race on his own but is willing to bet if Stu or Mike share the bet with him. Show why this behavior is consistent with David having von Neumann-Morgenstern utility function.

c) Let *w* denote wealth. In the finance literature a common model of decision making under uncertainty represents individuals as having a utility function of the form $E[U(w)] = f(\mu, \sigma^2)$, where μ denotes mean wealth and σ^2 the variance.

   One utility function that satisfies the above condition is the quadratic function. If $U(w)$ is a quadratic utility function then $E(u)$ depends only on μ and σ^2. Calculate the Arrow-Pratt index of absolute risk aversion for the quadratic function $U(w) = a + bw + cw^2$. Discuss the merits of quadratic utility functions in light of your results.

d) It is often observed that individuals under-insure irreplaceable items (for example, an item given to them from a family member). Is this behavior consistent with expected utility theory? Explain why or why not.

QUESTION B2: "INSURANCE"

Consider the market for health insurance. Suppose that the market is comprised of four groups of people of differing risk categories. There are a large and equal number of people in each group, but insurers cannot tell which group a person belongs to (i.e. this is a situation of asymmetric information). Each group faces a risk of requiring medical treatment of value $10,000.

Suppose that the willingness to pay of people in each group is as follows:

Agent	A	B	C	D
Risk	0.2	0.4	0.6	0.8
Willingness to pay	2,500	5,200	6,800	8,500

a) Find the actuarial fair premium and the risk premium for each group.
b) Suppose now that the risk category is private information. What premium would an insurance company have to charge to break even? Will all agents participate?
c) Continuing with the logic from b), what will be the price of insurance in the equilibrium and which groups will participate?
d) Suppose participation were mandatory under the private information scenario. Show whether or not this is an efficient outcome from a societal perspective and explain your answer.

PART C: Choice. Answer either C1 or C2. If you answer both, only C1 will be graded.

QUESTION C1: "BRIDGING THE GAP"

A bridge could be built across a river that currently has no passage. The fixed costs (in terms of interest charges on a permanent loan to finance construction of the bridge) are \$3060 per week and variable costs are \$0.10 per crossing. The bridge has a capacity of 1800 crossings per week and is uncongested up to that point. The demand function of the representative consumer is;

$$q = 15 - 20p + .2M$$

where p is the price per crossing (measured in dollars) and M is income. There are 200 of these typical consumers with incomes of \$150 each.

In all cases, show your work and annotate your steps and answers.

a) Assume this project is a public project. The planner wants to assure that the bridge is utilized, but there is no excess demand. What price and quantity are optimal?
b) What is the *maximum* amount each consumer would be willing to pay, over and above the cost to use the bridge in order for it to be built?
c) If a private, profit-maximizing, entrepreneur were to build the bridge, what price would be charged?
d) Would the private entrepreneur build the bridge?

QUESTION C2: "AN ALL CONSUMING QUESTION"

Please show all of your steps and annotate your answer. Consider a consumer with an expenditure function of $e(p_x, p_y, U) = 2(p_x p_y U)^{0.5}$ - which is consistent with a utility function of $U(x, y) = xy$.

In all cases, show your work and annotate your steps and answers.

a) Find this consumer's uncompensated demand (Marshallian demand) for both *x* and *y*.
b) Suppose initially that p_x = \$4 per unit, p_y = \$5 per unit and the consumer's income, *M*, is \$100. The price of *x* then increases to \$6 per unit. Would a subsidy of \$15 be enough to make the consumer as well off as before the price change?
c) For the original price and income conditions (p_x = \$4, p_y = \$5 and *M* = \$100) find the values of the income and substitution effects. From this information, is this a normal good? Explain.
d) This consumer's preferences are complete, transitive, and reflexive because they can be represented on a real number line. Demonstrate whether or not this consumer's preferences are continuous and exhibit non-satiation and diminishing marginal rates of substitution.