**Ph.D. MICROECONOMICS CORE EXAM** **August 2008**

This exam is designed to test your broad knowledge of microeconomics. There are three sections: one required and two choice sections. You must complete both problems in the required section and one choice problem in each of the two choice sections, giving you a total of five problems to complete during the allotted time. The required problems are in section A and the choice problems are in sections B and C. If you should answer more than one choice question in a section, only the first will be considered.

IMPORTANT. You are expected to adhere to the following guidelines in completing the exam for your answer to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. Number all pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number.
- Provide clear, concise discussion to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Explain your steps in any mathematical derivations. Simplify your final answers completely.

**PART A**

**All of the problems in Part A (A1, A2 and A3) are required. Answer all parts of all questions.**

**QUESTION A1**

A household consists of two individuals who are both potential workers and who pool their budgets. The preferences are represented by a single utility function $U(x_0, x_1, x_2)$ where $x_1$ is the amount of leisure enjoyed by person 1, $x_2$ is the amount of leisure enjoyed by person 2, and $x_0$ is the single, composite consumption good enjoyed by the household. The two members of the household have $T_1, T_2$ hours per week, respectively, which can either be enjoyed as leisure or spent on paid work. The hourly wage rates for the two individuals are $w_1, w_2$, respectively and they jointly have non-wage income of $\tilde{y}$, and the price of the composite good is unity.

a) Develop the budget constraint for the household and explain it.

b) Suppose the household utility function takes the form

$$U(x_0, x_1, x_2) = \sum_{i=0}^{2} \beta_i \log(x_i - a_i).$$

where $a_i$ and $\beta_i$ are parameters, such that, $a_i \geq 0$ and $\beta_i > 0$, with $\beta_0 + \beta_1 + \beta_2 = 1$.

Interpret these parameters. Solve the household's optimization problem and find the reduced form, Marshallian demand for consumption.

c) What are the labor supply functions for the two individuals.

d) Given your response in c), what is the response of an individual's labor supply to an increase in
   (i) his or her own wage,
   (ii) the other persons wage, and
   (iii) the non-wage income.

   Explain these results.

**QUESTION A2**

Consider a duopoly with identical firms. The cost function for firm $f$ is:

$$TC_f = C_0 + cq_f, f = 1,2,$$

and the inverse demand for the total industry output is:

$$P = \beta_0 - \beta q,$$

where $C_0$, $c$, $\beta_0$, and $\beta$ are all positive numbers. Total output in the industry is given by:

$$q = q_1 + q_2.$$

a) Find the isoprofit contour and the reaction function for firm 2.

b) Find the Cournot-Nash equilibrium for the industry and illustrate (and explain) it in $(q_1, q_2)$ space.

c) Find the cooperative solution for the industry and illustrate it on the same diagram (reproduced for this question) from b). Explain the change and the impact on firm profits.

d) If firm 1 acts as a leader and firm 2 as a follower, find the solution to the game. Draw the set of payoffs for the three solutions.

**<u>PART B</u> Answer question B1 or B2. If you complete more than one problem, only the first will be considered.**

**QUESTION B1**

Ed Bull works in a china shop and can choose to be either careful or careless (act like "a bull in a china shop"). If he is careful, there is a 50% chance that he will break some china. If he is careless, the chance of breaking some china rises to 75%. If Ed breaks china, he will be fired and have no wealth; but if he avoids breaking any china, he will keep his job and have a wealth of W. Ed dislikes being careful and values being careless by E, a lump sum of utility that is added to his utility of wealth function. He has a strictly concave utility of wealth function. Now let us assume that an insurance company decides to sell unemployment insurance to Ed. If he is fired, this insurance will restore his wealth to W.

(a) Discuss the concept of Moral Hazard and how it relates to this problem.

(b) Suppose that the insurance company can observe Ed's actions. Thus, if any china is broken, the company will know whether Ed was careful or careless. Assume that $\pi$ is the cost of each unit of insurance (that is, it costs $\pi$ to insure 1 unit of wealth W). If $u(W/2) > u(W/4) + E$, show that an insurance contract that sets $\pi = 1/2$ when Ed is careful and $\pi = 3/4$ when Ed is careless will lead him to be careful and buy full insurance coverage (that is, buy W units of insurance for $\pi W$).

(c) Now, suppose that the insurance company cannot observe Ed's actions. As a result, if any china is broken, the company will not know whether Ed's carelessness caused the accident. Determine the optimal insurance contract in this situation.

**QUESTION B2**

George inherits an amount of money $x_0$ that is placed in a bank account at time 0. If left in the bank, the money will earn interest at rate *r*. George, who fashions himself a trust-fund baby plans to live off of his inheritance. That is, Georges only source of income for consumption in any time period, $c(t)$, is from this bank account. George, knowing that you are an economist, wants you to develop a consumption path that maximizes lifetime utility, where his utility is defined by

$$u = \int_0^T e^{-\rho t} U(c(t))dt,$$

where the interval of time $(0, T)$, is George's lifetime, $U(c(t))$ is the instantaneous utility from consuming an amount $c(t)$ at time *t*, and $\rho \geq 0$ is George's personal rate of time preference. George's utility function is $U(c(t)) = \ln c(t)$. A requirement of George's inheritance is that he leaves a bequest of money in the bank account of pre-determined amount, *b*.

a) Set up George's optimization problem, identify the objective function, the transition equation, and any constraints. Form the current value Hamiltonian for this problem where $\mu(t)$ is the current value shadow value (adjoint variable).

b) Since George is not real swift with numbers and equations make his head spin, he wants you to show him your results in picture form. Luckily, the system of differential equations to solve this problem are linear so you can find an explicit solution, which you can graph. Solve for $c(t)$ and provide two graphs that show the time paths for $c$ and $x$ when $\rho > r$ and $\rho < r$. Explain the difference in these two outcomes to George.

c) What happens when $\rho = r$?

**PART C Answer question C1 or C2. If you complete more than one problem, only the first will be considered.**

**QUESTION C1**

Suppose you have to pay \$2 for a ticket to enter a competition. The prize is \$19 and the probability that you win is $\frac{1}{3}$. You have an expected utility function with $u(x) = \ln x$ and your current wealth is \$10.

(a) Define and calculate the certainty equivalent of this competition?

(b) Define and calculate the risk premium?

(c) Should you enter the competition? Explain.

**QUESTION C2**

Given a firm's cost minimization problem:

$$\begin{aligned} &\min TC = wL + rK \\ &s.t \quad x = AK^{\alpha} L^{\beta} \end{aligned}$$

(a) Find the conditional input demands. Why are they called "conditional"?

(b) Show that $TC^* = \left\{ \frac{1}{A} \left[ \left( \frac{\beta}{\alpha} \right)^{\alpha} + \left( \frac{\alpha}{\beta} \right)^{\beta} \right] \right\}^{\frac{1}{\alpha+\beta}} (r)^{\frac{\alpha}{\alpha+\beta}} (w)^{\frac{\beta}{\alpha+\beta}} (x)^{\frac{1}{\alpha+\beta}}$

(c) Your goal is to estimate the cost function in (b), so you linearize it by taking the natural log and add a random (normal) error term. Discuss the economic meaning and possible restrictions on the following expression derived from the econometric model:

$$\frac{\text{coefficient for ln w}}{\text{coefficient for ln x}} + \frac{\text{coefficient for ln r}}{\text{coefficient for ln x}}$$