**Ph.D. MICROECONOMICS CORE EXAM** **August 2010**

This exam is designed to test your broad knowledge of microeconomics. There are three sections: one required and two choice sections. You must complete both problems in the required section and one choice problem in each of the two choice sections, giving you a total of four problems to complete during the allotted time. The required problems are in section A and the choice problems are in sections B and C. If you should answer more than one choice question in a section, only the first will be considered.

IMPORTANT. You are expected to adhere to the following guidelines in completing the exam for your answer to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number all pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number.**
- Provide clear, concise discussion to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Explain your steps in any mathematical derivations. Simplify your final answers completely.

## PART A

**Both problems in Part A (A1 and A2) are required. Answer all parts of all questions.**

**QUESTION A1**

Each of the following statements is True, False, or Uncertain (neither always true or always false). Evaluate each statement as True, False, or Uncertain and give a brief but complete explanation for each answer, using mathematical derivation where necessary. Credit depends on your explanation.

A. Consider a **monopolist** that uses two inputs, $x_1$ and $x_2$, with respective input prices $w_1$ and $w_2$. For this firm, $\frac{\partial x_1}{\partial w_2} < 0$.

B. Consider the special case of a utility function that is additively separable, that is $U(x) = \sum_i U(x_i)$. If all goods exhibit diminishing marginal utility then between any two goods, the marginal rate of substitution will be diminishing.

**QUESTION A2**

Consider the following one-shot, incomplete-information Cournot game. (Recall that the Cournot model is where two firms simultaneously choose output levels and the market-clearing price depends on each firm's output level.)

It is public information that the inverse market demand is $P = 750 - Q$ (where $Q = q_1 + q_2$). It is also public information that there are no fixed costs and firm 1's marginal costs are \$100/unit. Additionally firm 2 knows its own marginal costs, however firm 1 does not know what firm 2's marginal costs are *(*this is a form of the incomplete information*)*. Firm 1 believes that there is a 30% chance that firm 2's marginal cost is \$200/unit *(high cost)* and a 70% chance that firm 2's marginal cost is \$50/unit *(low cost).* Assume that the objective of both firms is to maximize expected profit.

A. What are the Bayesian Nash equilibrium quantities for this game?

B. What is the profit for each firm in each case (firm 2 having a high marginal cost and firm 2 having a low marginal cost)? What is the expected profit for firm 1 (considering the probability of each event happening)?

C. Now, a law firm offers that, with its legal services, it could eradicate the private information. This means that firm 2 would know firm 2's cost structure **before** it makes its quantity decision. Note that the probability of firm 2 being a particular type before the law firm is hired will stay the same (30% *high cost* and a 70% *low cost).* What is the maximum firm 1 would be willing to pay for the law firm's services? (Again assume that both firm's objective is to maximize expected profits.)

**PART B**

**Answer question B1 or B2. If you complete more than one problem, only the first will be considered.**

**QUESTION B1**

A. A direct utility function is given by $U(x_1, x_2) = \alpha \ln x_1 + x_2$. Use Roy's identity to construct demand functions for the two goods and compare with the demand functions derived from the direct utility function.

B. Use Shepard's lemma to find the production function *f* that corresponds to the cost function $C(w_1, w_2, y) = (w_1 + 2\sqrt{w_1 w_2} + w_2)y$. Demonstrate that f is a Constant Elasticity of Substitution (CES) production function (general form $q = A[\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho}]^{-1/\rho}$).

## QUESTION B2

Suppose $U(x_1,x_2) = x_1 + \sqrt{x_2}$. The respective prices of the two goods are $p_1$ and $p_2$ and the individual has income $m$.

A. Construct Marshallian demand curves for this utility function and interpret.

B. Verify the Slutsky Equation for the case of $\frac{\partial x_2}{\partial p_1}$ and interpret.

**C.** Consider a price change for good $x_2$. Show that $CV(p_2^0, p_2^1, m) = EV(p_2^0, p_2^1, m)$.

**<u>PART C</u>**

**Answer question C1 <u>or</u> C2. If you complete more than one problem, only the first will be considered.**

**QUESTION C1**

Big Tony owns a restaurant and wants to hire hard-working waiters. He has interviewed Sophie who currently works at a shop down the street for $20,000 and no tips. Sophie's shop-girl job requires no effort. Sophie would consider working for Big Tony, but waiting tables imposes costs on Sophie of $t^2/200$ where t= the number of tables served.

Big Tony's offer to Sophie includes a base salary of *$x* plus tips. Tips at Big Tony's are reliably $10 per table. Big Tony wants to make sure that Sophie works hard, and so payment of her base salary will only happen if Sophie serves at least 1,000 tables that year. If Sophie serves less than 1,000 tables she keeps only her tips.

A. What must Big Tony offer in order to hire Sophie away from her current position?

B. How many tables will Sophie serve in a year?

C. Given the number of tables Sophie serves and the amount of money Big Tony will pay, how much does Sophie cost Big Tony on a per-table basis? Remember that Big Tony does not pay the tip portion of Sophie's earnings.

D. Does the cultural norm of tipping help or hurt Big Tony? Sophie? Explain with reference to your approach and findings in the sub-parts above.

**QUESTION C2**

Pat's utility function is given as $U(w)=50w-2w^2$.

A. Derive at least one measure of Pat's risk aversion.

B. Pat's current wealth is 10. Pat has been offered the following 2 deals:

   Deal 1: 50% chance that Pat's final wealth will be 15 and 50% chance it will be 2.

   Deal 2: 50% chance that Pat's final wealth will be 12 and 50% chance it will be 3.

   Pat CANNOT choose to just keep the current wealth of 10 – Pat must pick Deal 1 or 2. Which deal should Pat choose?

C. Explain your result with particular attention to characteristics of Pat's utility function. How do increases in wealth affect Pat's utility? Is there a reasonable story you could tell that would support this utility function?