Ph.D. MICROECONOMICS CORE EXAM
August 2013

This exam is designed to test your broad knowledge of microeconomics. There are three sections: one required and two choice sections. You must complete both problems in the required section and one choice problem in each of the two choice sections, giving you a total of four problems to complete during the allotted time. The required problems are in section A and the choice problems are in sections B and C. If you should answer more than one choice question in a section, only the first will be considered.

IMPORTANT. You are expected to adhere to the following guidelines in completing the exam for your answer to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number all pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number.**
- Provide clear, concise discussion to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Explain your steps in any mathematical derivations. Simplify your final answers completely.
- When you turn in your exam answers double check to make sure you have included all the pages to each question number, in order. The pages you submit as your answer are the only ones that will be considered.
- To simplify copying, please leave 1 inch borders.

PART A: REQUIRED QUESTIONS

Both problems in Part A (A1 and A2) are required. Answer all parts of all questions.

QUESTION A1

Consider a principal agent problem in which an agent has a utility function equal to $u(w, a) = w^{.75} - a$, where w indicates the wage paid by the principal to the agent and a indicates the cost of effort exhibited by the agent. In this problem the agent can either exhibit "high" effort, in which case $a = 200$ or "low" effort in which case $a = 100$. Also, if the agent doesn't accept this job, her next best alternative is a job that pays \$500 and requires no effort ($a = 0$). The principal has the option to either hire or not hire the agent; however, the level of effort exhibited by the agent is not observed by the principal.

a) First, assuming that $a = 0$, provide the agent's Arrow-Pratt measure of absolute risk aversion. Compare this agent with an agent that has a utility function equal to $u(w, a) = \ln(w)$. Which agent is more risk averse?

b) Now, assume that if the job is accepted and the agent exhibits low effort there will be a 0.4 probability that she will receive \$500 and a 0.6 probability that she will receive \$2000. Alternatively, if she exhibits high effort there is a 0.2 probability of receiving \$500 and a 0.8 probability of receiving \$2000. Will the agent exhibit high effort, low effort, or not accept the job at all?

c) In this section the principal is trying to design the optimal contract in which the agent both (1) accepts the contract and (2) works hard. Recall that the principal doesn't observe the effort level of the agent. However, in this section the principal does observe the performance of the company (good or bad), which is correlated with the agent's effort. Specifically, if the agent exhibits low effort there is a 0.5 probability that the performance of the company will be good and a 0.5 probability that the performance of the company will be bad. Alternatively, if the agent exhibits high effort the probability of there is a 0.8 probability that the performance of the company will be good and a 0.2 probability that the performance of the company will be bad. Assume that the principal will pay the agent \$500 if the company performs poorly. What is the optimal transfer payment for a profit-maximizing principal to offer the agent (when the performance of the company is good), which will induce the agent to both accept the contract and exhibit high effort?

d) Re-examine section c) only this time assume that the principal *does observe* the effort level exhibited by the agent. Again, assume that the principal will pay the agent \$500 if the company performs poorly (although this very well might not be the case). What will be the optimal contract in which the agent both accepts the contract and works hard? Explain the logic of this contract and how being able to determine the agent's action improves or hurts the principal's negotiating power.

QUESTION A2

There are two goods, food and clothing, whose quantities are denoted by F and C and prices by p and q, respectively. Our sole consumer, Ted, has income denoted by I and the utility function $U(F,C) = F^{0.5}C^{0.5}$. Initially $I = 100$, $p = 1$, and $q = 1$. Suppose the price of food increases to $p' = 1.21$. Income and the price of clothing do not change. (Note: In the questions that follow, you must derive all required functions.)

a) How much extra income would the government need to give Ted to restore him to his original utility level?

b) Calculate the reduction in Ted's consumer surplus when the price of p increases.

c) Compare the compensating variation (calculated in (a)) and the change in consumer surplus (calculated in (b)) and explain why the two are the same or different.

PART B: CHOICE QUESTIONS

Answer all parts of either question B1 or B2. If you complete more than one problem, only B1 will be considered.

QUESTION B1

Consider the utility function $U(x_1, x_2) = (x_1)^{0.5} + x_2$

a) Restricting prices to those that yield interior solutions, find the first-order conditions for maximizing the utility function subject to a budget constraint, $p_1x_1 + p_2x_2 = y$, and check to see that second-order conditions are satisfied.

b) Without restricting prices to those that yield interior solutions, find the first-order conditions for maximizing the utility function subject to the budget constraint.

c) Restricting prices to those that yield interior solutions, derive the indirect utility function dual to the utility function and verify Roy's Identity.

d) Restricting prices to those that yield interior solutions, find the first-order conditions for minimizing expenditure $p_1x_1 + p_2x_2$ subject to a utility constraint, $U(x) = u$, and check to see that second-order conditions are satisfied.

QUESTION B2

a) Let $\succcurlyeq$ be represented by $U: \mathbb{R}^n_+ \rightarrow \mathbb{R}$. Prove that if U(x), where **x** is a vector, is strictly increasing then $\succcurlyeq$ is strictly monotonic. In a few sentences, explain the implications of strict monotonicity of preferences.

b) Set up a general cost minimization problem. Use the Envelope Theorem to provide an economic interpretation of the multiplier.

c) Mr. Tri Angle consumes only three goods. An economist has collected the data shown in the following on Mr. Tri Angle's consumption behaviour. Does Mr. Tri Angle's behavior satisfy the weak axiom of revealed preference?

observation	p1	p2	p3	x1	x2	x3
1	1	2	3	3	2	1
2	2	1	3	3.5	2	0.5
3	2	2.25	1	2	3	1

PART C: CHOICE QUESTIONS

Answer all parts of either question C1 or C2. If you complete more than one problem, only C1 will be considered.

QUESTION C1

Consider the following international trade game in which there are three types of agents: (1) domestic firms, (2) foreign firms, and (3) a domestic government. In this game the government moves first setting a per-unit tariff (which is represented by t) that is charged to each *foreign* firm. After the domestic government has chosen the per unit tariff rate, the domestic and foreign firms move at the same time, choosing how much quantity to produce. In this game there are n_f foreign firms and n_d domestic firms. q_f is the quantity produced by the foreign firm and q_d is the quantity produced by the domestic firm, and the industry has an inverse demand curve equal to $P(Q) = \propto - Q$ (where $Q = q_f n_f + q_d n_d$). Firms are profit maximizers, the cost structure for domestic firms is $c(q_d) = c_d q_d$ and the cost structure for foreign firms is $c(q_f) = c_f q_f$. The government maximizes social welfare, which is equal to $W = .5Q^2 + n_d \pi_d + n_f q_f t$.

a) First assume that there are no foreign firms ($n_f = 0$) and one domestic firm ($n_d = 1$). What quantity will the domestic firm produce and what price will they charge? What will the deadweight loss be in this industry? What will the social welfare be in this industry?

b) Next assume that there is one foreign firm ($n_f = 1$), one domestic firm ($n_d = 1$) and a domestic government. What will the optimal tariff be? How much profit will each firm make? What will the domestic social welfare for this industry?

c) Instead in this section assume that ($n_d = 0$) and that, more generally there are n_f foreign firms. What is the optimal tariff? How much quantity will be provided in this market?

QUESTION C2

In "Affirmative action in college admissions: Examining labor market effects of four alternative policies." *Contemporary Economic Policy* (2002), Bruce Wydick used an incomplete information, signaling game to evaluate the impact of four alternative affirmative action strategies. Specifically, he used a two-player game with one player being an employer and the other player being a student. In this game the student's objective is to maximize wages less the cost of attending college. The student can have a number of types: he/she can be gifted, represented by g (with probability γ) or mediocre, represented by m (with probability $(1-\gamma)$). Also, the student can either be "advantaged," represented by a (with probability α) or "underprivileged," represented by u (with probability $(1-\alpha)$). So, to be clear a student can have four distinct types (gifted-advantaged, gifted-underprivileged, mediocre-advantaged, and mediocre-underprivileged). In this game the student observes his/her type and chooses one of two actions (college or no college). The employer observes this action choice (but not the student's type) and chooses whether to hire the student as a manager or as a mailroom clerk. The benefit that an employer gets from hiring a gifted employee is represented by q_g and the benefit the employer from hiring a mediocre employee is represented by q_m, where $q_g > w > q_m$. Furthermore, it is assumed that the productivity of all types working in the mailroom is ε, which is slightly greater than zero.

a) First, represent this games using extensive form, making sure to include all pertinent types, probabilities, and payoffs in your game tree. Next, generally explain the role of signaling in the context of education. As formally as possible explain what employers trying to accomplish and why education may be a good mechanism to accomplish this.

b) To understand this signaling game in the context of affirmative action, Wydick provides specific parameters to consider. First, he assumes that the wage that the employer pays to a manager is equal to 1 and he normalizes the wage paid to a mailroom clerk to zero. Also, he assumes that the cost of a gifted type attending college is equal to .4 and that the cost of a mediocre type attending college is equal to .8 (i.e. $c_g = .4$ and $c_m = .8$). Additionally, he considers two types of disadvantage levels. Specifically, he considers a *heterogeneous* disadvantage level, where the cost of an advantaged type attending college is .4 and the cost of an underprivileged type attending college is .9 (i.e., $c_a = .4$ and $c_u = .9$). Alternatively, he considers the possibility that there is a *homogeneous* disadvantage level, which he defines as $c_a = .7$ and $c_u = .8$. First, provide all pure strategy, Bayesian Nash equilibria when there is *heterogeneous* disadvantage level. Next, provide all pure strategy Bayesian Nash equilibria when there is *homogeneous* disadvantage level. Finally, explain the difference between the equilibria of these two different models. (with all equilibria make sure to describe your notation and in this section assume that $\gamma q_g + (1-\gamma)q_m < 1$)

c) Wydick considers the impact of race based preferential admissions. Specifically, he models the result of a race based preferential admission that would reduce the cost of attending college equal to $R = .25$. Provide the new Baysian Nash Equilibria given this reduction in psychic costs for both the *homogeneous* and *heterogeneous* cases. Explain how this reduction is the psychic cost of admission changed these equilibria. It should now be clear why Wydick distinguishes between homogeneous and heterogeneous disadvantage. Explain and discuss. (In this section assume that $\gamma q_g + (1-\gamma)q_m < 1 + \varepsilon$)