

**Ph.D. MICROECONOMICS CORE EXAM**  
**August 2014**

This exam is designed to test your broad knowledge of microeconomics. There are three sections: one required and two choice sections. You must complete both problems in the required section and one choice problem in each of the two choice sections, giving you a total of four problems to complete during the allotted time. The required problems are in section A and the choice problems are in sections B and C. If you should answer more than one choice question in a section, only the first will be considered.

IMPORTANT. You are expected to adhere to the following guidelines in completing the exam for your answer to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number all pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number.**
- Provide clear, concise discussion to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Explain your steps in any mathematical derivations. Simplify your final answers completely.
- When you turn in your exam answers double check to make sure you have included all the pages to each question number, in order. The pages you submit as your answer are the only ones that will be considered.
- To simplify copying, please leave 1 inch borders.

**PART A: REQUIRED QUESTIONS**

Both problems in Part A (A1 and A2) are **required**. Answer all parts of all questions.

**QUESTION A1**

Assume that there are two identical firms, firm 1 and firm 2, who are producing a product  $q$ . The inverse demand curve for  $q$  is expressed as  $p(q_1, q_2) = 200 - q_1 - q_2$ . Both firms have a constant marginal cost of \$10.

- a. First, find the Cournot output, price, and profit (label this  $\pi_{cor}$ ) for each firm. Next, if each firm produces half of the collusion market output, find the collusion output, price, and profit for each firm (label this profit  $\pi_{col}$ ). Third, a firm can deviate from collusion and use its reaction function to find profit maximizing output given that the other firm keeps colluding. Given this, find the defecting firms output and profit (label this  $\pi_{def,cop}$ ), the cooperating firms output and profit (label this  $\pi_{cop,def}$ ), and the market price. Finally, assume that when the two firms cooperate, they get collusion profits; when the two firms defect, they get Cournot profits; and if one firm defects and the other cooperates, they get payoffs found in part c. Fill in table 1 using the firm's profit for cooperating and defecting.

		Firm 2	
		Cooperate (C)	Defect (D)
Firm 1	Cooperate (C)	$\pi_{col}, \pi_{col}$	$\pi_{cop,def}, \pi_{def,cop}$
	Defect (D)	$\pi_{def,cop}, \pi_{cop,def}$	$\pi_{cor}, \pi_{cor}$

Table 1

- b. What is the one-shot simultaneous Nash equilibrium in Table 1?
- c. Now assume the game is infinitely repeated. For what rate of return will a player defect *once*, if the other player is playing a tit-for-tat (TFT) punishment strategy?
- d. Now assume a possibility exists such that **the game will continue next period** with probability  $p$ . What is the minimum effective rate of return for which cooperation remains the equilibrium against a player that uses a defect forever grim trigger strategy? How does an increase in  $p$  affect this result?

## QUESTION A2

Use the following production functions to answer the questions below. Denote the price of labor  $w$ , the price of capital  $r$ , and output as  $y$  with an output price of  $p$ .

(a)  $f(L, K) = 2L + 3K$

(b)  $f(L, K) = L^{1/3}$

- a. Derive the factor demand functions for  $L$  and  $K$  associated with (a) and (b) above.
- b. Derive the cost functions associated with (a) and (b).
- c. Assuming  $w = r$ , determine whether a unique profit-maximizing level of output exists. If it does, state the profit-maximizing level of output.
- d. Assuming  $w = r$ , at what level of output are the two technologies equally profitable? At higher levels of output, which technology is more profitable?

## **PART B: CHOICE QUESTIONS**

**Answer all parts of either question B1 or B2. If you complete more than one problem, only B1 will be considered.**

### **QUESTION B1**

Today grocery store customers can pay for their groceries using either traditional cashiers with cash registers or self-checkout machines with a managing employee. Suppose both technologies are represented by Leontief production functions. Assume one cashier per cash register for Technology A and a one employee per six self-checkout machines for Technology B. Denote the price of labor  $w$ , the price of capital  $r$ , units of labor as  $L$  and units of capital as  $K$ , where either a cashier or employee equals one unit of  $L$  and a cash register or a self-checkout machine equals one unit of  $K$ .

- a. Write out the production functions for grocery checkout services for each separate technology.
- b. Assuming that the two technologies are interchangeable from a customer's perspective, what is the profit-maximizing firm's cost function across both technologies?
- c. In the real world, one typically sees stores using both types of technologies, i.e., the two technologies are not perfectly interchangeable. Assuming an elasticity of substitution of one between the two types of technologies, and decreasing returns to scale, write out an appropriate production function across the two technologies,  $f(A, B)$ , which allows for a profit-maximizing solution in which both technologies are used.
- d. Suppose customers that prefer Technology A have a higher willingness-to-pay overall. How much more would a given customer have to be willing to pay to use Technology A in order for the firm to be indifferent between using the two technologies?

## QUESTION B2

Suppose an agent's preferences are represented by  $u(x,y)=x^2 + y^2$ . Assume prices and income and strictly positive.

- a. Find the Marshallian Demand
- b. Find the Indirect Utility
- c. Is this utility function consistent with the typical assumptions made about preferences? Why or why not?

## **PART C: CHOICE QUESTIONS**

**Answer all parts of either question C1 or C2. If you complete more than one problem, only C1 will be considered.**

### **QUESTION C1**

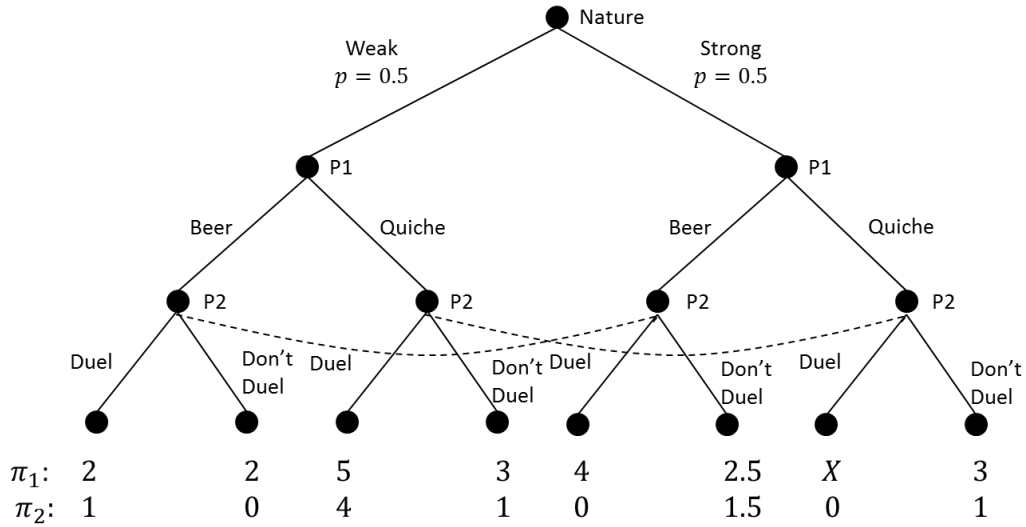
Consider an economy with two producing sectors: fishing ( $f$ ) and recreation ( $r$ ). Let all consumers in the economy be identical and thus characterized by a single representative consumer who derives utility from consuming all two of the produced goods. The consumer's well behaved utility function is given by  $U = U(f, r)$  and is increasing in each of the two arguments.

The consumer is endowed with labor ( $\bar{L}$ ). They receive no utility from consumption of their labor and sell it **all** to producing sectors (labor stock is fully employed). Production in all sectors is characterized by quasi-concave production functions. Production in the fishery is defined by the technology  $f = F(l_f)$ , where  $l_f$  is labor in the fishery and where the stock of fish is taken as a constant and omitted from the problem. Fishing has consequences on other species as it removes fish and lowers the food available for whales. The whale stock is thus a decreasing function of fishing such that  $W(f)$  and  $W_f < 0$ . Whales in turn are an important input to recreation (i.e. whale watching) through its production function  $r = R(l_r, W(f))$ , where  $R_l > 0$  and  $R_W > 0$ .

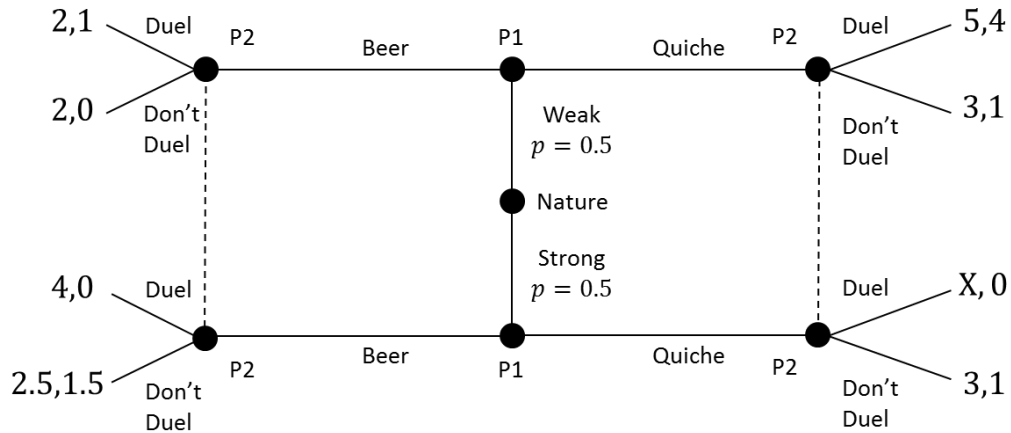
- a. Derive and interpret the Pareto efficient outcome.
- b. Derive and interpret the competitive outcome.
- c. Why do (a) and (b) not coincide? Relate your findings to what you know about the first fundamental theorem of welfare economics.
- d. Demonstrate (and derive) how the implementation of a tax on the fishing industry will provide the correct incentives in the economy such that the competitive outcome will achieve the Pareto optimal outcome.

**QUESTION C2**

Use either of the extensive form games below to answer the following questions (same game, just different representation):



OR



Profits:  $(\pi_1, \pi_2)$

Let  $X = 2$ :

- Does a pure strategy separating Bayesian Nash equilibrium exist? If so, derive and interpret. If not, explain why and who deviates.
- Does a pure strategy pooling Bayesian Nash equilibrium exist? If so, derive and interpret. If not, explain why and who deviates.

Let  $X = 4$ :

- Does a pure strategy separating Bayesian Nash equilibrium exist? If so, derive and interpret. If not, explain why and who deviates. How is this different from  $X = 2$ ?
- Does a pure strategy pooling Bayesian Nash equilibrium exist? If so, derive and interpret. If not, explain why and who deviates. How is this different from  $X = 2$ ?