

**Ph.D. MICROECONOMICS CORE EXAM**  
**January 2015**

This exam is designed to test your broad knowledge of microeconomics. There are three sections: one required and two choice sections. You must complete both problems in the required section and one choice problem in each of the two choice sections, giving you a total of four problems to complete during the allotted time. The required problems are in section A and the choice problems are in sections B and C. If you should answer more than one choice question in a section, only the first will be considered.

IMPORTANT. You are expected to adhere to the following guidelines in completing the exam for your answer to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number all pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number.**
- Provide clear, concise discussion to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Explain your steps in any mathematical derivations. Simplify your final answers completely.
- When you turn in your exam answers double check to make sure you have included all the pages to each question number, in order. The pages you submit as your answer are the only ones that will be considered.
- To simplify copying, please leave 1 inch borders.

## **PART A: REQUIRED QUESTIONS**

**Both problems in Part A (A1 and A2) are required. Answer all parts of all questions.**

### **QUESTION A1**

Suppose that there is a single representative agent that owns all firms and has preferences over three goods:  $l$ , leisure,  $x$ , a clean good, and  $y$ , a dirty good. Suppose that the production of the clean good is negatively impacted by the level of production of the dirty good. At this point assume that there is a fixed quantity of labor,  $L$ , supplied to the labor market and that the consumer has no choice of the quantity of leisure,  $l$ . In principle one could suppress the leisure choice at this point.

- a. Derive and interpret the socially (Pareto) optimal solution to the problem.
- b. Derive and interpret the competitive solution to the problem. Explain the difference between part a. and b. A graph could be useful here.
- c. Derive and interpret the optimal tax needed to restore the Pareto solution in a competitive world.
- d. Offer an alternative to a tax system that would also restore optimality.

## QUESTION A2

Suppose you are a baker, who makes pita bread. (Note: the pita bread industry is highly competitive.) In order to make one order of pita bread, you need ingredients (A, S, Y, O, H, and L) in the following proportions:

- 1 pound all-purpose flour (A)
- 2 teaspoons salt (S)
- 1.5 teaspoons instant yeast (Y)
- 1 tablespoon olive oil (O)
- 1 cup water (H)
- 1 hour labor (L)

A is measured in pounds, S and Y in teaspoons, O in tablespoons, H in cups, and L in hours. You also pay a variety of fixed costs, which include rent, the oven, measuring utensils, baking sheets, and bowls, which cost a total of “F”. (Assume electricity and gas are included in your rent, i.e., electricity and natural gas are a variable cost for your landlord and a fixed cost for you.)

- a. Derive the *short-run* production function and *short-run* cost function for the pita baker.
- b. Suppose that you discover you can substitute whole wheat flour (W) for all-purpose flour in a one-to-one ratio. Now what does the short-run production function look like?
- c. Suppose that you can substitute non-instant yeast for instant yeast. Non-instant yeast costs half as much as instant yeast, but requires twice the number of hours to cook, which doubles labor costs. What would the labor/yeast price ratio have to be for it to be more profitable to use the instant yeast? From a real world perspective, which type of yeast are you most likely to use?
- d. Specify your long-run cost function. Under what conditions would you choose to temporarily shut-down? When would you choose to exit the industry?

## **PART B: CHOICE QUESTIONS**

**Answer all parts of either question B1 or B2. If you complete more than one problem, only B1 will be considered.**

### **QUESTION B1**

Suppose  $U=2X_1^{0.5}X_2^{0.5}$ . Both  $X_1$  and  $X_2$  are normal goods.

- a. Illustrate graphically CV, EV, and change in CS if  $P_1^0$  decreases to  $P_1^1$ .
- b. Suppose  $P_1^0=\$2$ ,  $P_2^0=\$2$ , and income is \$100.  $P_1$  decreases such that  $P_1^1=\$1$ . Calculate CV, EV, and change in CS. (You may assume positive  $X_1$  and  $X_2$  and that the SOCs hold.)
- c. Suppose there was a monotonic transformation of utility such that  $V=2U=4X_1^{0.5}X_2^{0.5}$ . Without redoing your calculations explain whether CV, EV, and the change in CS increase, decrease, or stay the same.
- d. Identify and explain a situation in which  $CV=EV=\text{change in CS}$ .

## QUESTION B2

Suppose a consumer has preferences represented by the following utility function:

$$u(x_1, x_2; r_1) = (x_1 - r_1)^\alpha x_2^\beta$$

With  $\alpha + \beta = 1$ ,  $0 < \alpha < 1$ ,  $0 < \beta < 1$ , and  $r_1 \geq 0$ . Assume that for  $x_1 < r_1$  or  $x_2 < 0$ , utility is equal to zero. Good 1 is an addictive good with addiction level  $r_1$ . The more you have consumed the addictive good in the past (the more of an addict you are), the higher  $r_1$  becomes.

- Is it better (in terms of total utility) to be a little bit of an addict (small  $r_1$ ) or a serious addict (large  $r_1$ )? How does the marginal utility of  $x_1$  change as  $r_1$  changes? Explain why your answers make sense.
- Use the standard budget constraint to determine the optimal level of consumption of  $x_1$  and  $x_2$  as a function of  $p_1, p_2, r_1, m, \alpha$ , and  $\beta$ . In equilibrium, do the “barely” addicted (low  $r_1$ ) or serious addicts (high  $r_1$ ) consume more of Good 1? What about Good 2?
- Explain under what conditions/assumptions preferences represented by this type of utility function are rational and satisfy local non-satiation.

## **PART C: CHOICE QUESTIONS**

**Answer all parts of either question C1 or C2. If you complete more than one problem, only C1 will be considered.**

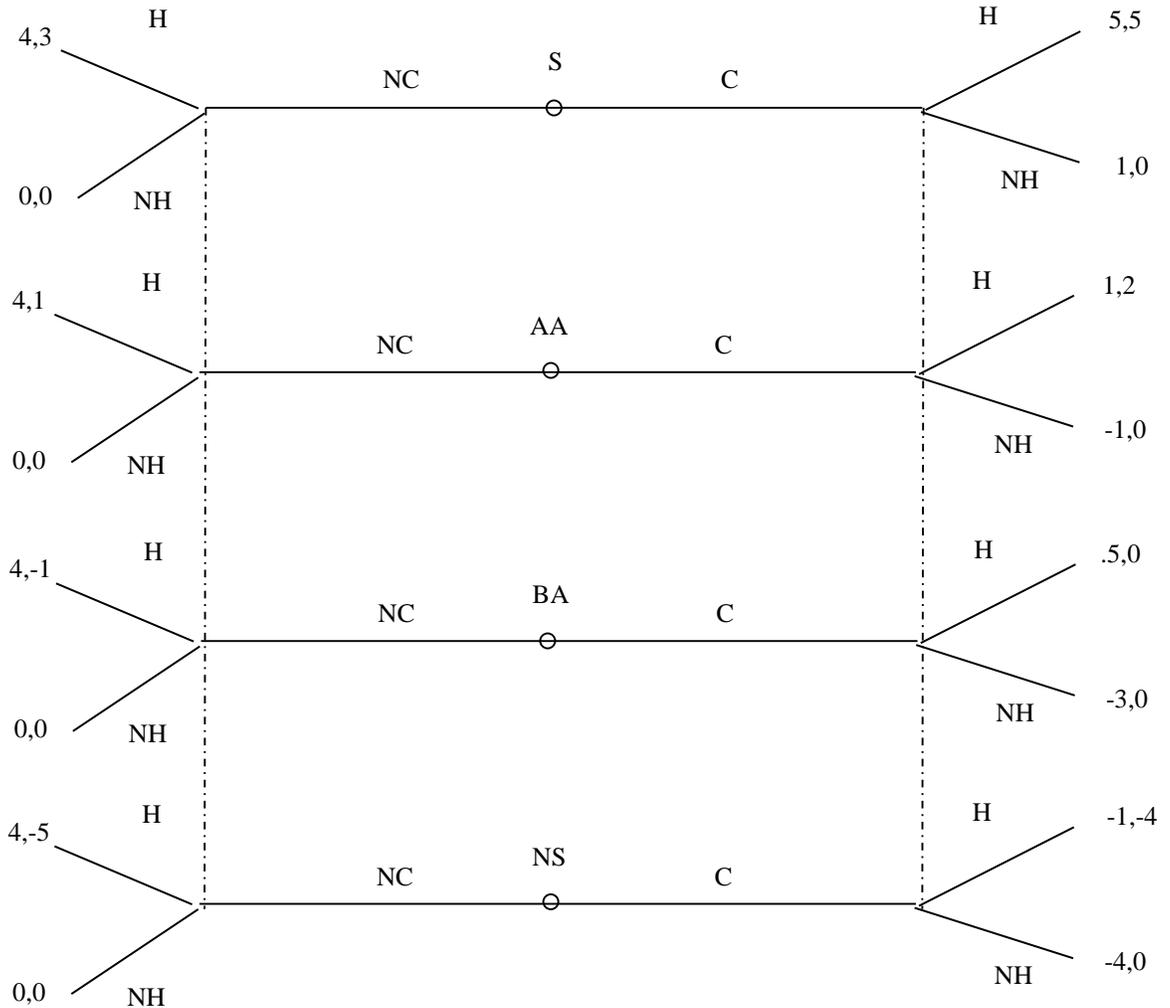
### **QUESTION C1**

Pigiver (of the three little pigs) builds a house, how many nails should he use? The problem is that nails are expensive to purchase, but Pigiver wants a strong house to withstand the Big Bad Wolf, and strength requires a lot of nails. Suppose Pigiver's preferences over wealth can be represented by a von Neumann-Morgenstern utility function, and that Pigiver has an initial wealth of  $W$ . The probability the Wolf can blow Pigiver's house down is  $p$ . The cost of nails is given by  $C(n)$ , where  $n$  is the number of nails. The damage done to the house if the wolf blows the house over is a function of the number of nails used,  $D(n)$ . Both  $C(n)$  and  $D(n)$  are twice continuously differentiable. Assume Pigiver is risk neutral.

- a. Pigiver is an expected utility maximizer; set up his problem and characterize how Pigiver chooses the optimum number of nails. That is, interpret the first-order condition, which may require assumption on the above functions.
- b. What assumptions will ensure the solution is a maximum?
- c. How would a change in the probability of a hurricane or a change in Pigiver's initial wealth affect the number of nails he uses?
- d. How does your answer to a and b change if the probability that the wolf can blow the house down is endogenous? Specifically, you can assume that the number of nails has a negative effect on the probability, where  $p(n), p' < 0$ .

## QUESTION C2

Consider the following signaling game. In this game the sender can be one of four potential types: smart (S), above average (AA), below average (BA), and not smart (NS). These types are determined by nature and only observed by the sender. The sender observes his/her type and then chooses a message  $M = \{NC, C\}$ . Once the receiver observes this message she chooses to either hire (H) or not hire (NH) the sender. **In all parts describe your notation.**



- Describe the payoffs for this game. Does college improve productivity? Is college costly? Do these payoffs make sense? Explain
- Assume that  $p(S)=.2$ ,  $p(M)=.3$ ,  $p(BA) =.3$  and  $p(D)=.2$ . What are the pure strategy NE for this game? Discuss.
- Keeping the same probabilistic assumptions (from b) now assume that the government implements a college scholarship program effectively reducing the cost of college by 2. What are the new pure strategy NE? Discuss.
- Describe the differences in the equilibria in b from the equilibria in c. Is this a good social program? Does this model represent what you think would happen in the real world? Explain.