Ph.D. CORE EXAM
August 2021

This exam is designed to test your broad knowledge of microeconomics and macroeconomics. You must complete all four problems during the allotted time.

IMPORTANT. You are expected to adhere to the following guidelines in completing the exam for your answer to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number all pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number.**
- Provide clear, concise discussion to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Explain your steps in any mathematical derivations. Simplify your final answers completely.
- When you turn in your exam answers double check to make sure you have included all the pages to each question number, and in order. The pages you submit as your answer are the only ones that will be considered.
- To simplify copying, please leave 1 inch borders.

Question 1

Amy spends her income on two goods coffee (x_1) and burrito (x_2), with prices p_1 and p_2. Her preferences are given by the following utility function:

$$u(x_1, x_2) = {x_1}^{0.25}{x_2}^{0.75}$$

a) Find Amy's Marshallian demand functions for coffee and burrito.

b) Find Amy's expenditure function using duality. Verify Roy's identity.

c) Suppose Amy's income is $1000. The price of burrito is $3. The price of coffee is $1 if she is a loyalty member of the coffee house and $2 if she is not a member. The loyalty membership program costs $100. Find Amy's optimal consumption bundle if she is a member. Find Amy's optimal consumption bundle if she is not a member.

d) Should Amy purchase the membership? Explain your answer.

Question 2

Economic Growth and Government Investments in Infrastructure

Consider the Ramsey model of an economy in competitive equilibrium. There is a representative household and a representative firm. The household's utility functional is

$$U \equiv \int_0^\infty u(c_t)e^{-\rho t}dt,$$

with

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta},$$

where $1 > \rho > n = 0$, and $\theta > 1$.

The representative firm has a production function

$$F[K_t, G_t, L_t] = AK_t^\alpha (G_t L_t)^{1-\alpha},$$

where G_t is the total quantity of infrastructure provided by the government in this economy at time t. Further, assume infrastructure grows at the constant rate γ. Thast is,

$$\dot{G}_t = \gamma G_t$$

For simplicity, Assume capital does not depreciate after production ($\delta = 0$). Find the competitive equilibrium of this economy, using the following steps.

a) Write down the representative household's maximization problem, solve it, and derive the 4 equations that characterize the solution.

b) Write down the firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form. Derive the 2 equations that characterize the solution.

c) What are the equilibrium conditions for this economy?

d) Combine your answers to parts a) - c) and derive a pair of differential equations for the variables c and k. Can you draw a phase diagram? If you can't draw a phase diagram, can you transform the differential equations in order to be able to draw a phase diagram? If so, carefully identify and derive mathematically all the important points. Is there a balanced growth path? What is the growth rate of the economy?

e) Do the following comparative dynamics exercise: $\gamma' > \gamma$. As always, assume you are starting at the steady state at the time of the change in γ. Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of the logs of c and k for both cases. Discuss.

Question 3

Consider a 2×2 exchange economy. Consumer 1's utility function is $U_1(x_1, y_1) = 2\sqrt{x_1} + 4\sqrt{y_1}$ and her endowment is $\omega_1 = (4, 1)$. Consumer 2's utility function is $U_2(x_2, y_2) = x_2 + 8\sqrt{y_2}$ and his endowment is $\omega_2 = (4, 5)$.

a) Derive the contract curve (the set of Pareto optimal allocations) for this economy (please specify the entire set of points that make up the contract curve). Draw the contract curve in an Edgeworth box.

b) Set up the optimization problem for each consumer and derive the offer curves (demand functions for x_1, y_1, x_2, y_2).

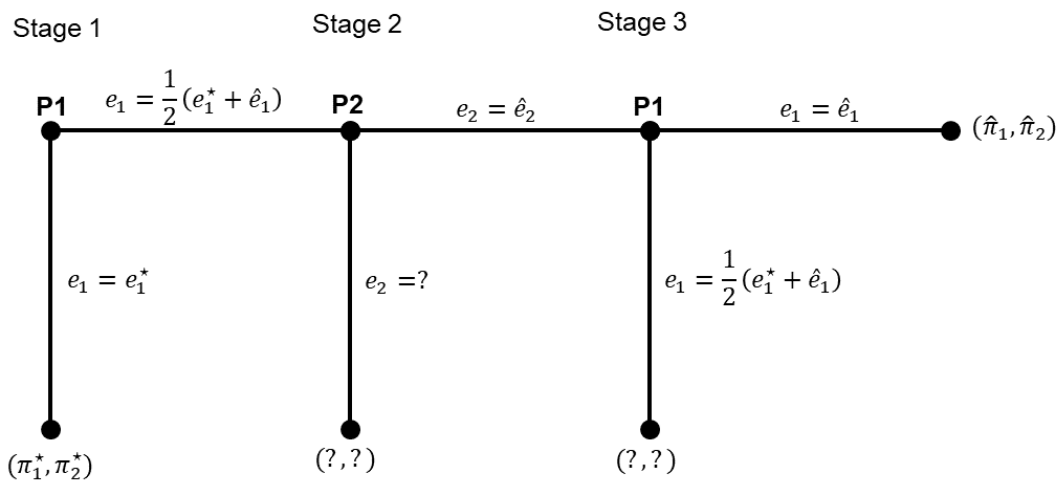
c) Find the competitive equilibrium for this economy. This will be a price vector $p^\star$ together with an allocation $(x^\star, y^\star)$. Confirm that this outcome is Pareto optimal, and show it on your figure.

d) Now suppose that most of consumer 2's endowment is destroyed by a natural disaster, such that they are left with only $\omega_2{}' = (0, 1)$. [Note: there is no change to consumer 1's endowment.] What is the new competitive equilibrium price and allocation with this endowment?

Question 4

Two people are working together to produce a good X. The effort each puts forth increases the quantity that is produced. The amount produced is $X = \alpha e_1 + \alpha e_2 + \left(\frac{1}{\alpha}\right) e_1 e_2$, where e_1 and e_2 are effort levels by each person, and α is a parameter that reflects how successively they can cooperate, with $\alpha > 0$. They will sell X for price $p = 1$, and split the revenue equally. Effort is costly for each person, with total cost of effort for each individual is represented by $C_i(e_i) = e_i^2$. [Note, the people split the revenue equally, but each pays his or her own cost.]

a) Solve for the optimal effort levels that maximizes the sum of profits for the two individuals (as a function of α). Call these effort levels $\hat{e}_i$. What are effort levels and profits when $\alpha = 1$?
b) If both people choose their effort levels simultaneously, solve for the Nash equilibrium effort levels (as a function of α). Call these effort levels $e_i^\star$. What are the effort levels and profits in this situation when $\alpha = 1$?
c) Now suppose the two individuals will play a three stage game as described in the figure below [Note: despite the three stages, production occurs only once according to the effort levels at whichever terminal node is reached]. Throughout this part <u>assume $\alpha = 1$</u>.



- Stage 1: Player 1 goes first. He can either play $e_1 = e_1^\star$ (*down*) or $e_1 = \frac{(e_1^\star + \hat{e}_1)}{2}$ (*right*). If he plays *down*, then player 2 responds with $e_2 = e_2^\star$ and then the game ends (outcome from part b). If player 1 plays *right*, then he is committing to this effort level and the game continues into the next stage.
- Stage 2: Player 2's move. She can play *down*, and choose an effort level that is a best response to $e_1 = \frac{e_1^\star + \hat{e}_1}{2}$ [Note: this is a sequential move game because player 1 has committed to an effort level], and end the game; or she can play *right*, which commits her to $e_2 = \hat{e}_2$, and the game continues into the next stage.

- Stage 3: Player 1's move. He can play *down*, and keep his effort level at $e_1 = \frac{e_1^\star + \hat{e}_1}{2}$ (and $e_2 = \hat{e}_2$ and the game ends), or he can play right and change his effort level to $e_1 = \hat{e}_1$ (and $e_2 = \hat{e}_2$ and the outcome is the same as part a).

Calculate the numerical values for all effort levels in the game, and calculate the payoffs at each terminal node (you can re-draw the game and write in the values). Then solve for the subgame perfect Nash equilibrium strategies and identify the outcome of the game.

d) Finally, suppose that there is uncertainty about the true value of α. Nature will either select $\alpha = 1/2$ or $\alpha = 3/2$, and each is equally likely. Player 1 knows what nature selects. Player 2 knows the probabilities that Nature will select each value of α, but not what was selected. Players 1 and 2 will choose their effort levels simultaneously. Solve for the Bayesian Nash equilibrium effort levels.