Ph.D. Economics Core Exam
August 2022

This exam is designed to test your broad knowledge of microeconomics and macroeconomics.

From the graduate student handbook:

"…the examination is not limited to the student's course work, but tests a student's grasp of economics as a whole. Questions require the student to demonstrate a superior grasp of the theory and the tools. The student's performance is evaluated by the manner in which they approach the problem; their demonstrated understanding of economic theory and application; and their ability to interpret results."

You must complete all **four** problems during the allotted time.

Note on Testing Procedure: You are expected to adhere to the following guidelines in completing the exam for your answers to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number ALL pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number**.
- Provide clear, concise discussion and economic intuition to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Fully explain your steps in any mathematical derivations. Simplify your final answers completely.
- When you turn in your exam answers double check to make sure you have included all the pages to each question number, and in order. The pages you submit as your answer are the only ones that will be considered.
- To simplify copying, please leave 1-inch borders.

Question #1

Suppose all consumers in an economy have the same Cobb-Douglas utility function:

$$u(g,x) = g^a x^{1-a}$$

where g denotes gasoline and x denotes all other goods (think of x as a composite good). Suppose the income level of consumer i is m_i.

a) Find Marshallian demands for consumer i. Find indirect utility function.

b) Suppose in 2021, The prices for both g and x are \$1. Total consumer income (M) in the economy was \$1 billion. Of which, \$200 million was spent on gasoline and \$800 million was spent on all other goods. Use this information to find a. Use the value you obtain in the remainder of the problem.

c) Now suppose that the government implements a subsidy to gasoline. For each unit gasoline sold, the government subsidizes the consumer \$0.1. This effectively reduces the gasoline price to \$0.9. The price of x and total consumer income stay unchanged. Find the total amount of the subsidy paid out.

d) Now suppose instead of directly subsidizing price, the government grants a \$500 subsidy to each consumer. Suppose Ada (a consumer) is indifferent between the two subsidy plans. That is, Ada's utility levels are the same when the government subsidizes gasoline directly and when the government implements a cash transfer. Find Ada's income level.

Question #2

Consider the Ramsey model of an economy in competitive equilibrium with technological growth. There is a representative household and a representative firm. The household's utility functional is

$$U \equiv \int_0^\infty \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt,$$

with where $1 > \rho > n = 0$.

The representative firm has a production function $F[K_t, A_t, L_t] = K_t^\alpha (A_t L_t)^{1-\alpha}$, and

$$\dot{A}_t = \phi A_t$$

where A_t is the level of technology that grows at the constant rate ϕ. Assume no capital depreciation ($\delta = 0$). Find the competitive equilibrium of this economy, using the following steps.

a) Write down representative household's maximization problem, solve it, and derive the 4 equations that characterize the solution.

b) The firm's first-order conditions for this problem are (I am giving them to you, no need to derive them):

$$\alpha A_t^{1-\alpha} k_t^{\alpha-1} = R_t$$

$$(1-\alpha) A_t^{1-\alpha} k_t^\alpha = w_t$$

c) What are the equilibrium conditions for this economy?

d) Combine your answers to parts a) - c) and derive a pair of differential equations for the variables c and k. Can you draw a phase diagram?

e) Define $\widehat{k} = \frac{k}{A}, \widehat{y} = \frac{y}{A}$, and $\widehat{c} = \frac{c}{A}$. Derive a pair of differential equations for the variables $\widehat{c}$ and $\widehat{k}$. Can you draw a phase diagram? If yes, draw the phase diagram, carefully identifying all the important points and dynamics in the diagram's regions. Discuss.

f) What is the growth rate of the economy? Show it.

g) Do the following comparative dynamics exercise: $\phi' < \phi$. As usual, the baseline economy starts in its balanced growth path (steady state in hats) at time $t = 0$. The modified economy starts at time $t = 0$. Remember that capital is the state variable. Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of the logs of c and k for both cases.

h) Go back to part d) and use the pair of differential equations for the variables c and k. Using those, assume now $\alpha \to 1$. Can you draw a phase diagram? If yes, draw the phase diagram, carefully identifying all the important points and dynamics in the diagram's regions. *Carefully discuss.*

Question #3

Consider a 2×2 exchange economy. Consumer 1's utility function is $U_1(x_1, y_1) = \left(\sqrt{x_1} + \sqrt{y_1}\right)^2$ and her endowment is $\omega_1 = (1, 1)$. Consumer 2's utility function is $U_2(x_2, y_2) = \ln(x_2) + y_2$ and his endowment is $\omega_2 = (4, 3)$.

a) Derive the contract curve (the set of Pareto optimal allocations) for this economy (please specify the entire set of points that make up the contract curve). Draw the contract curve in an Edgeworth box. Specify the initial endowment.

b) Set up the optimization problem for each consumer and derive the offer curves (demand functions for x_1, y_1, x_2, y_2).

c) Find the competitive equilibrium for this economy. This will be a price vector $p^\star$ together with an allocation $(x^\star, y^\star)$. Confirm that this outcome is Pareto optimal, and show it on your figure.

d) Now suppose there is a distribution of the initial allocation such that the new allocation is $\omega_1' = (5, 3)$ and $\omega_2' = (0,1)$. Find the new competitive equilibrium. Is this equilibrium Pareto optimal?

Question #4

<u>Note for this question only:</u> parts a) and b) are to be solved separately from parts c) and d). That is, parts a) and b) are unrelated to parts c) and d).

a) Suppose an investor has initial wealth w. She can choose to invest all or part of her wealth into the stock market. There are two possible rates of returns from the stock market: r_1 with probability p and r_2 with probability $1-p$, where $-1 < r_1 < 0 < r_2 < 1$. Also assume that $pr_1 + (1-p)r_2 > 0$. Denote her investment amount as M. (Hint: If the rate of return is r_1, her investment in the stock market will now be worth $(1+r_1)M$. The investor's utility function is $u(w) = \ln(w)$. Find the investor's expected utility.

b) Find the investor's optimal investment amount. Will she put more or less money into the stock market as her initial wealth increases?

Note: Recall, parts a) and b) are unrelated to parts c) and d) below.

c) There is a firm (call it firm 1) acting as a monopolist. There is a higher cost firm (call it firm 2) that is entering the market. The inverse demand function in this market is $P(Q) = a - bQ$, where Q is the total quantity produced (either by firm 1 alone, or by both firms 1 and 2 if firm 2 enters). Firm 1 has constant marginal cost of c_1 and firm 2 has constant marginal cost of c_2, with $c_2 > c_1 > 0$. Solve for the quantity produced when firm 1 is a monopolist (call this q_m). Separately, if firm 2 enters the market and the two firms choose their quantities simultaneously, solve for the Nash equilibrium quantities (call this $\hat{q}_1$ and $\hat{q}_2$). Also, solve for the profits of each firm in the Nash equilibrium ($\hat{\pi}_1$ and $\hat{\pi}_2$).

d) Now suppose that firm 2 has entered the market, and the firms will play an infinitely repeated game. The firms have decided to cooperate such that firm 1 chooses $q_1 = \frac{5}{8} q_m$ and $q_2 = \frac{3}{8} q_m$ (where q_m is the monopoly quantity from part c). The firms will each play these quantities if both firms have cooperated in all previous periods. If either firm deviates from cooperation in any previous period, then firms will play the quantities $\hat{q}_1$ and $\hat{q}_2$ (the Cournot strategies from part c) in all future periods. The discount factor between periods is $\delta = 1/2$. Is there a subgame perfect Nash equilibrium in which both firms play cooperation in each period? Demonstrate why or why not this equilibrium exists. The parameters are $a = 10$, $c_1 = 2$, and $c_2 = 3$.