Ph.D. Economics Core Exam
August 2023

This exam is designed to test your broad knowledge of microeconomics and macroeconomics.

From the graduate student handbook:

"…the examination is not limited to the student's course work, but tests a student's grasp of economics as a whole. Questions require the student to demonstrate a superior grasp of the theory and the tools. The student's performance is evaluated by the manner in which they approach the problem; their demonstrated understanding of economic theory and application; and their ability to interpret results."

You must complete all **four** problems during the allotted time.

Note on Testing Procedure: You are expected to adhere to the following guidelines in completing the exam for your answers to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number ALL pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number**.
- Provide clear, concise discussion and economic intuition to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Fully explain your steps in any mathematical derivations. Simplify your final answers completely.
- When you turn in your exam answers double check to make sure you have included all the pages to each question number, and in order. The pages you submit as your answer are the only ones that will be considered.
- To simplify copying, please leave 1-inch borders.

1) Consumer Theory

John, an employee at Facebook, resides in San Francisco (SF). His utility can be represented by the quasi-linear utility function:

$$U(x, y) = x + \sqrt{y},$$

where x represents the consumption bundle of non-durable goods (e.g., food and gas), and y represents his apartment (assuming divisibility). The prices for x and y are denoted as p_x and p_y, respectively. John's income is denoted as M.

a) Calculate John's Marshallian demand functions for goods x and y in terms of prices p_x and p_y, and his income M.

b) Determine John's indirect utility function and derive his expenditure function using duality.

c) In SF, the average price for the bundle of non-durable goods is $\$1,000$, and the average price for an apartment is $\$4,000$. John's income is $\$10,000$. During the pandemic, John relocated to Albuquerque (ABQ) to be closer to his family. In ABQ, the average price for the bundle of non-durable goods and an apartment is \$800 and $\$1,000$, respectively. If John aims to maintain the same level of utility he had in SF, how much money would he save?

d) With the amount of money John saved (as calculated in part c)), how does his consumption of x and y change compared to if he were to spend his entire income while living in ABQ? Discuss the relationship between this change and John's utility function.

2) Game Theory

Two people are working together to produce a good X. The effort (e_i) each puts forth increases the quantity that is produced, however, person 1's effort increases production more than person 2's effort. The amount produced is

$$X = 2e_1 + \frac{7}{10}e_2 + \frac{5}{2}e_1e_2,$$

where e_1 and e_2 are effort levels by persons 1 and 2, respectively. They will sell X for price $p = 1$, and split the revenue equally. Effort is costly for each person, with the total cost of effort for each individual represented by $C_i(e_i) = e_i^2$. [Note, the people split the revenue equally when they work together, but each pays his or her own cost of effort.]

a) However, before working together, they want to know what their profits would be if they worked independently. For person 1, she can produce the product according to $X(e_1) = 2e_1$, and for person 2, he can produce the product according to $X(e_2) = \frac{7}{10}e_2$. Calculate the optimal effort levels if each person works independently (i.e., each produces their own quantity of X, receives the full revenue from selling X, and pays their own cost of effort). Call these effort levels, $\tilde{e}_i$, and calculate their profits $\tilde{\pi}_i$.

b) Now both people will work together. If both people choose their effort levels simultaneously, solve for the Nash equilibrium effort levels. Call these effort levels $e_i^\star$. What are the effort levels and profits ($\pi_i^\star$) for each individual? [Don't forgot that they each get $1/2$ of the revenue from selling X].

c) Suppose person 1 gets to choose their effort level first. After person 1 chooses e_1, person 2 will observe this effort level and then choose their own effort level (e_2). Solve for the subgame perfect Nash equilibrium effort levels for both people (call these $\hat{e}_1$ and $\hat{e}_2$). Calculate the profits ($\hat{\pi}_i$) for each individual. Compare the outcomes (both in terms of the effort levels and profits) between the answers from parts a, b and c.

d) Person 1 does not think that this arrangement is fair to her because she is more productive than person 2. To remedy this, the two people will play a three-stage game, wherein the first stage, person 1 gets to choose the share, s, of the revenue that will go to person 1 (with $(1 - s)$ share of the revenue going to person 2). Specifically, person 1 can choose $s = 1/2$ (as it was in parts b and c) or she can choose $s = 4/5$. Person 2 will observe person 1's stage 1 choice. Stages 2 and 3 are played as in part c, with person 1 choosing her effort level in stage 2, and then person 2 observes person 1's effort level and then chooses their own effort level in stage 3. Solve for the subgame perfect Nash equilibrium strategies for both people (which includes the share chosen by person 1 in stage 1).

3) Upstream Technology Firm

Consider an industry with an inverse demand curve of

$$P = 30,000 - 25Q$$

where $Q = \sum_{i=1}^{n} q_i$. In this game there are t periods and n downstream firms. Downstream firms move simultaneously and compete over quantity. Each firm has a marginal cost of production ($c \geq 0$) equal to \$5,000 per unit and a discount rate equal to .9 (e.g., a payoff one period in the future period is worth 90% of a payoff today). In this game, there is also an upstream technology firm that has just invented a new production process that decreases the marginal cost of production to \$1,000. Assume, for this problem that cost structures will stay consistent for every year of the game (*i.e.*, if a firm does not have the technology its marginal cost will be \$5,000 for every period and if they have the technology its marginal cost would be \$1,000 for every period). The upstream firm moves first and offers a take-it-or-leave-it offer to only 1 firm to purchase this technology for a one-time lump sum price of F.

a) First, assume that $t = 1$ and $n = 1$. What is the most that the (downstream) firm would pay for the technology?

b) Next, assume that $t = \infty$ and $n = 1$. How much would the (downstream) firm pay for the technology?

c) Next, assume that $t = 1$ and $n = 2$. How much would the (downstream) firm pay for the technology?

d) Next, assume that $t = \infty$ and $n = 2$. How much would the (downstream) firm pay for the technology?

4) Financial Intermediation in a Risk Neutral Environment

Consider the Diamond-Dybvig model with two assets. There are three periods: $t = 0, 1, 2$. Agents are ex-ante identical, and are endowed with 1 unit of a single good at $t = 0$, and nothing at $t = 1, 2$. At the beginning of $t = 1$, and after any investment decision is made, a fraction $\pi = 1/2$ of agents learn that they prefer to consume only at $t = 1$, while the remaining fraction $(1 - \pi) = 1/2$ of agents prefers to consume only at $t = 2$. There is a linear production technology whereby one unit of the good invested in period 0 yields $R = \mathit{2}$ units of the good at time 2. This technology is illiquid, in the sense that an investment that is interrupted in period 1 generates $r = \mathit{1/2}$ units of consumption at $t = 1$. In addition, there is a liquid storage technology, whose return is equal to 1 in both periods. Agents preferences are given by

$$u(c_1, c_2) = \left[\pi \frac{c_1^{1-\theta} - 1}{1-\theta} + (1-\pi)\frac{c_2^{1-\theta} - 1}{1-\theta}\right]$$

with $\theta = 0$.

a) Write down the problem of an agent in autarky, the FOC, and solve for the optimal consumption vector (c_1^a, c_2^a). What is the expected utility? Carefully discuss.

b) Now suppose that in period 1, after agents learn their idiosyncratic consumption preference shock and before they consume, a financial market opens where agents can trade claims for the returns on the illiquid production technology. Let p be the price of a bond that yields one unit of the illiquid production technology at $t = 2$. Write down the problem of an agent in this setting. What will the equilibrium price of a bond be in this case (and why)? What is the consumption vector(c_1^b, c_2^b)? What is the expected utility? Carefully discuss.

c) Now suppose agents form coalitions, which they call non-profit credit unions, and pool their resources. Write down the problem of an agent in this setting, the FOC, and solve for the optimal consumption vector (c_1^{cu}, c_2^{cu}) and the expected utility.

d) Compare a)-c). Graph the solutions, including budget constraints, and carefully discuss.

e) Can multiple equilibria arise in the credit union environment? Why/Why not? If yes, what policies will prevent multiple equilibria (ie, prevent banking panics?). Carefully discuss.