Ph.D. CORE EXAM
October 2020

This exam is designed to test your broad knowledge of microeconomics and macroeconomics. You must complete all four problems during the allotted time.

IMPORTANT. You are expected to adhere to the following guidelines in completing the exam for your answer to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number all pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number.**
- Provide clear, concise discussion to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Explain your steps in any mathematical derivations. Simplify your final answers completely.
- When you turn in your exam answers double check to make sure you have included all the pages to each question number, and in order. The pages you submit as your answer are the only ones that will be considered.
- To simplify copying, please leave 1 inch borders.

Question 1

Consider a consumer with the following utility function:

$$u(x_1, x_2) = (x_1 - b)^{0.5} x_2^{0.5}$$

Market price level is denoted by $\mathrm{P} = (p_1, p_2)$. The income endowment of the consumer is y. $b > 0$.

a) Set up the consumer's expenditure minimization problem. Find Hicksian demands.

b) Find the indirect utility function using duality.

c) Find the consumer's Marshallian demands.

d) Assume that $y = \$200$ and $b = 2$. Suppose that the price level goes from (1,1) to (1,2). Calculate the compensating variation of the price change.

Question 2

Social Security Defined Contributions

Consider an economy consisting of an infinite sequence of two period lived, overlapping generations. N_t agents are born in period t, with $N_{t+1} = (1+n)N_t$. In each period there is a single final good that is produced using a constant returns to scale technology with capital and labor as inputs. Let k_t denote the time t capital-labor ratio, and let $f(k_t)$ denote the intensive production function. Let f have the Cobb-Douglas form $f(k_t) = Ak_t^{\alpha}$, with $0 < \alpha < 1$. One unit of the final good that is not consumed at t converts into one unit of capital at $t+1$. Capital depreciates after production, with $\delta \in (0,1)$. Agents have the utility function

$$u(c_{1,t}, c_{2,t+1}) = \frac{c_{1,t}^{1-\theta} - 1}{1-\theta} + (1+\rho)^{-1} \frac{c_{2,t+1}^{1-\theta} - 1}{1-\theta}$$

with $\theta \to 1$, and $\rho = 0$.

Suppose that, besides saving in assets (a_t), young agents born in period t are required by the government to contribute some defined amount to social security (label this variable d_t). These funds are invested in capital, and agents receive the return from their own social security defined contributions, given by $(1 + r_{t+1})d_t$, when they are old.

a) Write down the household's maximization problem and derive the equations that characterize the solution. Discuss.

b) Write down the firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form.

c) What are the equilibrium conditions for this economy? Pay particular attention to the savings=investments equilibrium condition.

d) Combine your answers to parts (a) - (c) and derive a *Law of Motion* (*LoM*) equation that defines a difference equation for the variable k. Looking at it, can we say anything about a steady-state solution? Can you graph the *LoM?*

e) Is the non-trivial steady-state in the Competitive Equilibrium (CE) Pareto Optimal (PO)? Carefully show and explain why, or why not. If not, how could you modify this government program to make it PO?

Question 3

Consider a 2×2 exchange economy. Consumer 1's utility function is $U_1(x_1, y_1) = \ln(4 + x_1) + \ln(y_1)$ and her endowment is $\omega_1 = (1, 1)$. Consumer 2's utility function is $U_2(x_2, y_2) = x_2 y_2$ and his endowment is $\omega_2 = (2, 0)$.

a) Derive the contract curve (the set of Pareto optimal allocations) for this economy (please specify the entire set of points that make up the contract curve). Draw the contract curve in an Edgeworth box.

b) Set up the optimization problem for each consumer and derive the offer curves (demand functions for x_1, y_1, x_2, y_2).

c) Find the competitive equilibrium for this economy. This will be a price vector $p^\star$ together with an allocation $(x^\star, y^\star)$. Confirm that this outcome is Pareto optimal, and show it on your figure.

d) Now suppose that the initial allocation is changed to $\omega_1' = (1, 0)$ and $\omega_2' = (2, 1)$. What is the new competitive equilibrium price and allocation with this endowment? Is this allocation Pareto optimal?

Question 4

Two people are working together to produce a good X. The effort each puts forth increases the quantity that is produced. The amount produced is $X = e_1 + e_2 + be_1e_2$, where e_1 and e_2 are effort levels by each person, and b is a parameter that reflects how successively they can cooperate. They will sell X for price $p = 1$, and split the revenue equally. Effort is costly for each person, with total cost of effort for each individual represented by $C_i(e_i) = \frac{1}{2}e_i^2$. [Note, the people split the revenue equally, but each pays his or her own cost.]

a) If both people choose their effort levels simultaneously, solve for the Nash equilibrium effort levels (as a function of b).

b) Now plug in the value $b = 1$, and calculate the effort levels by each, the total output produced, and the profits for each person. What happens to profits for each person when $b = 3/2$?

c) Now suppose that the people choose effort sequentially, and that $b = 1$. Person 2 will go first. Person 1 will observe person 2's effort and then make his decision. Solve for the subgame-perfect Nash equilibrium efforts. Calculate the output and profits for each. How does this compare with the effort levels from part b (when $b = 1$).

d) Finally, suppose that person 1 can be one of two types: either they are cooperative (C) or uncooperative (U). Person 1 knows what type they are, but person 2 does not. If person 1 is cooperative then the duo will work well together and $b = 3/2$. If person 1 is uncooperative, then they will struggle to work together and $b = 1/2$. Both people will make their effort level decisions simultaneously. Solve for the Bayesian Nash equilibrium effort levels.