Ph.D. Economics Core Exam
October 2022

This exam is designed to test your broad knowledge of microeconomics and macroeconomics.

From the graduate student handbook:

"…the examination is not limited to the student's course work, but tests a student's grasp of economics as a whole. Questions require the student to demonstrate a superior grasp of the theory and the tools. The student's performance is evaluated by the manner in which they approach the problem; their demonstrated understanding of economic theory and application; and their ability to interpret results."

You must complete all **four** problems during the allotted time.

Note on Testing Procedure: You are expected to adhere to the following guidelines in completing the exam for your answers to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number ALL pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number**.
- Provide clear, concise discussion and economic intuition to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Fully explain your steps in any mathematical derivations. Simplify your final answers completely.
- When you turn in your exam answers double check to make sure you have included all the pages to each question number, and in order. The pages you submit as your answer are the only ones that will be considered.
- To simplify copying, please leave 1-inch borders.

Question #1

Suppose a representative consumer, Nate, has the following utility function: $\mathrm{u}(x_1, x_2) = (x_1 - a_1)^{0.5}(x_2 - a_2)^{0.5}$. Nate's income is w and the prices for the two goods are p_1 and p_2, respectively. a_1 and a_2 are constants and are both greater than zero.

a) Find the Marshallian demand functions. Derive the indirect utility function.
b) Derive the Hicksian demand functions using Shephard's Lemma.
c) Assume that $a_1 = a_2 = 0$ for the remainder of the question. Suppose that the original prices of the two goods are ($1, $1). A supply shortage of good x_1 has increased its price to $2. The government plans to subsize the consumer by a one-time transfer payment $M. Find the smallest value of M to ensure that Nate maintains his original utility level.
d) Suppose that the government plans to issue the same $M to all consumers, based on your calculation in part c. There is another consumer, Kate, who has the same utility function as Nate. Kate's income is 2w, which is twice as high as Nate's income. Find the change in Kate's utility level with the price increase and the government transfer payment.

Question #2

Consider an economy consisting of an infinite sequence of two-period lived overlapping generations. In each period there is a single final good that is produced using a constant returns to scale technology with capital and labor as inputs. Let k_t denote the time t capital-labor ratio, and let $f(k_t)$ denote the intensive production function. Let f have the Cobb-Douglas form $f(k_t) = Ak_t^\alpha$, with $0 < \alpha < 1$. Capital fully depreciates after production ($\delta = 1$), and there is no population growth $N_{t+1} = N_t$, $n = 0$. Agents work when young only, can save assets labeled a_t, and have the utility function

$$u(c_{1,t}, c_{2,t+1}) = \frac{c_{1,t}^{1-\theta} - 1}{1-\theta} + (1+\rho)^{-1}\frac{c_{2,t+1}^{1-\theta} - 1}{1-\theta}$$

with $\theta \to 1$, and $\rho = 0$.

Assume the government imposes a tax τ on labor income of the young and uses these funds to subsidize old agents in the same period. In particular, for each unit of assets a_t owned by old born at time t, the government gives them a subsidy of σ_{t+1}. This is a pay-as-you-go social security retirement system, similar to what we have in the US, where the working young get a social security tax taken off their wages, and then receive social security payments when old.

a) Write down the household's maximization problem and derive the equations that characterize the solution $(a_t, c_{1,t}, c_{2,t+1})$.

b) The solution to the firm's maximization problem is given by [no need for you to derive it].

$$f'(k_t) = R_t$$

$$w_t = f(k_t) - k_t f'(k_t)$$

c) What are the equilibrium conditions for this economy? What is the government budget constraint?

d) Combine your answers to parts (a) - (c) and derive a *Law of Motion* (*LoM*) equation that defines a difference equation for the variable k. Get rid of all prices. Looking at it, can we say anything about a steady-state solution? Can you graph the *LoM?*

e) Is the non-trivial steady-state in the Competitive Equilibrium (CE) Pareto Optimal (PO)? Carefully show and explain why, or why not. Under what conditions will the CE be PO? Can you find an optimal tax, so that the CE is PO? What is the subsidy in this case? What is the optimal tax and subsidy combination when $\alpha = 1/3$? What is the optimal tax and subsidy combination when $\alpha = 1/4$?

f) Do the following comparative dynamics exercise. Initially, the CE economy is with $\sigma = \tau = 0$, and now the government imposes the optimal tax and subsidy rates that you found in part e), assuming $\alpha < 1/3$. Draw (i) LoM for both cases, indicating what is different, and (ii) the time paths of the logs of c and k for both cases.

Question #3

You are a homeowner (H) and your A/C has gone out in the middle of the summer. You call the repairperson (R) to fix your A/C. He identifies the problem and now you must determine whether to make the repairs and how much it will cost. The homeowner has a value, v, of having the A/C fixed, and the repairperson has a cost, c, of doing the work. Both H and R must submit a price for the job. Each can offer either a low price (L), medium price (M) or outrageous price (O) (where $O > M > L$). The price offered (or the strategy) by the homeowner and the repairman are p_H and p_R, respectively. If $p_H \geq p_R$ then the job will be done at $p = \frac{p_H + p_R}{2}$, and the payoff to H is $u_H = v - p$, and the payoff to R is $u_R = p - c$. Otherwise, the job will not be done and each gets zero payoff.

a) Assume both v and c are known by both H and R and the bids will be submitted simultaneously. Write out the normal form of this game.

b) Solve for the pure-strategy Nash equilibria from the game in part a in both of the following scenarios.

   i. $v > O$ and $c < L$

   ii. $v = M$ and $L < c < M$

c) Now suppose that the repairperson gets to make an offer first (same options as before: L, M, O) and then the homeowner responds with an offer (L, M, O). The price and payoffs are determined the same as above. Write out the extensive form of this game. Solve for all subgame perfect Nash equilibria for the following scenario (remember a strategy is an action at every information set)

   i. $M < v < O$ and $c < L$

d) Finally, go back to the simultaneous game in parts a and b. Suppose $L = 100$, $M = 200$, $O = 400$, $v = 250$, $c = 75$. Find all pure and mixed strategy equilibria of this game.

Question #4

<u>Note for this question only:</u> parts a) and b) are to be solved separately from parts c) and d). That is, parts a) and b) are unrelated to parts c) and d).

a) Consider a 2×2 exchange economy. Consumer 1's utility function is $U_1(x_1, y_1) = 2\ln(x_1 - 2) + \ln(y_1 - 1)$ and her endowment is $\omega_1 = (4, 2)$. Consumer 2's utility function is $U_2(x_2, y_2) = 4x_2 + \ln(y_2 - 4)$ and his endowment is $\omega_2 = (4, 4)$. Derive the contract curve (the set of Pareto optimal allocations) for this economy (please specify the entire set of points that make up the contract curve). Draw the contract curve in an Edgeworth box.

b) Set up the optimization problem for consumer 1 and derive her offer curve (demand function for x_1, y_1).

Note: Recall, parts a) and b) are unrelated to parts c) and d) below.

c) Suppose Ada's utility function is $u(w) = \sqrt{w}$, where w denotes her wealth. Assume that Ada owns a coffee shop. The coffee shop is worth $7,000 with a probability of 0.8. With a probability of 0.2, the shop will close and is worth $0. At what price would Ada be willing to sell her shop if she does not have any other asset? Now suppose that Ada received $500 from a friend as a gift. At what price would Ada be willing to sell her shop now?

d) Now assume that after receiving the $500 from her friend Ada can choose to spend the money to remodel her shop. Remodeling the shop would not change the value of the shop if it does not close but would change the probability that the shop closes. If she chooses to remodel, the probability that the shop closes would decrease from 0.2 to 0.15. Should Ada remodel the shop?