1) Consumer Theory

Consider the Quasi-linear utility function of the form:

$$\mathrm{U}(x_0, x_1, x_2) = cx_0 + f(x_1, x_2)$$

where the x_i are all market goods and c > 0 is a constant while the first derivative of $f(\cdot)$ with respect to x_i is also positive for i $= 1, 2$.

(a) Show that the utility maximizing conditions for the consumer demands will have no income effects for goods 1 and 2. Assume an interior solution (prices are such that the consumer will consume a positive amount of all goods).

For parts (b) to (e) assume $f(x_1, x_2) = a_1 \ln(x_1) + a_2 \ln(x_2)$ where $0 < a_i < 1$ for all i and $a_1 + a_2 < 1$, and still assume an interior solution to the utility maximization problem. Of course, this function for $f(\cdot)$ is part of the overall utility function that still involves x_0 as above.

(b) Find an equation for the demand curves for x_0 and for x_i for $i = 1, 2$.

(c) Find an equation for the indirect utility function from (b).

(d) Find an equation for the expenditure function from (b).

(e) Provide an equation or a quantitative result for the difference between Compensating and Equivalent variation based on this utility function.

2) Game Theory

Two firms produce a homogeneous good with a market inverse demand for the good of $P(Q) = 3 - Q/2$, where $Q = q_1 + q_2$, and q_i is the output for firm i. Each firm has costs of $C(q_i) = q_i$.

a. Assume that both firms choose production quantities simultaneously. Solve for the Nash equilibrium for this game.

b. Now assume that firm 1 chooses their quantity before firm 2. Solve for the subgame perfect Nash equilibrium in this game.

c. Now suppose that there are two stages to the game. In the first stage, each firm simultaneously chooses whether to commit to a quantity choice. Then, in the second stage, the firms play the following game. If both firms have committed to a quantity choice, the firms produce this quantity and price is determined by the inverse demand function. If neither firm commits, the standard Cournot game is played. If one firm commits to a quantity in stage one while the other does not, then the committed firm produces the quantity it has committed to and the rival firm chooses its quantity in response (like the game played in part b). To simplify the analysis, assume that the firms have only three possible moves in the first stage: they may commit to a quantity of 1, a quantity of 2, or they may choose not to commit. Find all pure strategy subgame perfect Nash equilibria for the two-stage game.

d. Finally, suppose the firms will play an infinitely repeated simultaneous quantity choice game. What range of discount factors can support the strategy of both firms choosing a quantity of 1 in every period? Potentially helpful equations: $\sum_{(t=0)}^{\infty} \delta^t = 1/(1-\delta), \sum_{t=1}^{\delta} \delta^t = \delta/(1-\delta)$.

3) Upstream Technology Firm

Consider an industry with an inverse demand curve of

$$P = 50,000 - 20Q$$

where $Q = \sum_{i=1}^{n} q_i$. In this game there are t periods and n downstream firms. Downstream firms move simultaneously and compete over quantity. Each firm has a marginal cost of production ($c \geq 0$) equal to \$5,000 per unit and a discount rate equal to .9 (e.g., a payoff one period in the future period is worth 90% of a payoff today). In this game, there is also an upstream technology firm that has just invented a new production process that decreases the marginal cost of production to $c = 0$. Assume, for this problem that cost structures will stay consistent for every year of the game (*i.e.*, if a firm does not have the technology its marginal cost will be \$5,000 for every period and if they have the technology its marginal cost would be \$0 for every period). The upstream firm moves first and offers a take-it-or-leave-it offer to only 1 firm to purchase this technology for a one-time lump sum price of T.

a) First, assume that $t = 1$ and $n = 1$. What is the most that the (downstream) firm would pay for the technology?

b) Next, assume that $t = \infty$ and $n = 1$. How much would the (downstream) firm pay for the technology?

c) Next, assume that $t = 1$ and $n = 2$. How much would the (downstream) firm pay for the technology?

d) Next, assume that $t = \infty$ and $n = 2$. How much would the (downstream) firm pay for the technology?

4) Financial Intermediation

Consider the Diamond-Dybvig model with two assets. There are three periods: $t = 0, 1, 2$. Agents are ex-ante identical, and are endowed with 1 unit of a single good at $t = 0$, and nothing at $t = 1, 2$. At the beginning of $t = 1$, and after any investment decision is made, a fraction $\pi = 1/2$ of agents learn that they prefer to consume only at $t = 1$, while the remaining fraction $(1 - \pi) = 1/2$ of agents prefers to consume only at $t = 2$. There is a linear production technology whereby one unit of the good invested in period 0 yields $R = 2$ units of the good at time 2. This technology is illiquid, in the sense that an investment that is interrupted in period 1 generates $r = 1/2$ units of consumption at $t = 1$. In addition, there is a liquid storage technology, whose return is equal to 1 in both periods. Agents preferences are given by

$$u(c_1, c_2) = \left[\pi \frac{c_1^{1-\theta} - 1}{1 - \theta} + (1 - \pi) \frac{c_2^{1-\theta} - 1}{1 - \theta} \right]$$

with $\theta \to \infty$.

a) Write down the problem of an agent in autarky, the FOC, and solve for the optimal consumption vector (c_1^a, c_2^a). What is the expected utility? Carefully discuss.

b) Now suppose that in period 1, after agents learn their idiosyncratic consumption preference shock and before they consume, a financial market opens where agents can trade claims for the returns on the illiquid production technology. Let p be the price of a bond that yields one unit of the illiquid production technology at $t = 2$. Write down the problem of an agent in this setting. What will the equilibrium price of a bond be in this case (and why)? What is the consumption vector(c_1^b, c_2^b)? What is the expected utility? Carefully discuss.

c) Now suppose agents form coalitions, which they call non-profit credit unions, and pool their resources. Write down the problem of an agent in this setting, the FOC, and solve for the optimal consumption vector (c_1^{cu}, c_2^{cu}) and the expected utility.

d) Compare a)-c). Graph the solutions, including budget constraints, and carefully discuss.

e) Can multiple equilibria arise in the credit union environment? Why/Why not? If yes, what policies will prevent multiple equilibria (ie, prevent banking panics?). Carefully discuss.