M.A. CORE EXAM
October 2021

This exam is designed to test your broad knowledge of microeconomics and macroeconomics. You must complete three out of the four problems during the allotted time.

IMPORTANT. You are expected to adhere to the following guidelines in completing the exam for your answer to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number all pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number.**
- Provide clear, concise discussion to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Explain your steps in any mathematical derivations. Simplify your final answers completely.
- When you turn in your exam answers double check to make sure you have included all the pages to each question number, and in order. The pages you submit as your answer are the only ones that will be considered.
- To simplify copying, please leave 1 inch borders.

Question 1

Suppose a consumer's utility function is $u(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}$. The prices of the two goods are p_1 and p_2, respectively. The consumer's total income is m.

a) Find the consumer's Marshallian demand functions.

b) Find the consumer's indirect utility function and expenditure function.

c) Suppose $p_1 = 1, p_2 = 2, m = 1000$. If a labor shortage caused p_1 to increase to $2, how much additional income does the consumer need to maintain her original utility level?

Question 2

Economic Growth and Government Investments in Infrastructure

Consider the Ramsey model of an economy in competitive equilibrium. There is a representative household and a representative firm. The household's utility functional is

$$U \equiv \int_0^\infty ln(c_t)e^{-\rho t}dt,$$

with where $1 > \rho > n = 0$, and $\theta \to 1$.

The representative firm has a production function

$$F\left[K_t, G_t, L_t\right] = AK_t^\alpha (G_t L_t)^{1-\alpha},$$

where G_t is the total quantity of infrastructure provided by the government in this economy at time t. Further, assume infrastructure grows at the constant rate γ. That is,

$$\dot{G}_t = \gamma G_t$$

Capital depreciates at after production at the constant rate $\delta > 0$. Find the competitive equilibrium of this economy, using the following steps.

a) Write down the representative household's maximization problem, solve it, and derive the 4 equations that characterize the solution.

b) Write down the firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form. Derive the 2 equations that characterize the solution.

c) What are the equilibrium conditions for this economy?

d) Combine your answers to parts a) - c) and derive a pair of differential equations for the variables c and k. Can you draw a phase diagram? If you can't draw a phase diagram, can you transform the differential equations in order to be able to draw a phase diagram? If so, carefully identify and derive mathematically all the important points. Is there a balanced growth path? What is the growth rate of the economy?

e) Do the following comparative dynamics exercise: $\gamma' < \gamma$. As always, assume you are starting at the steady state at the time of the change in γ. Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of the logs of c and k for both cases. Discuss.

Question 3

Consider a 2×2 exchange economy. Consumer 1's utility function is $U_1(x_1, y_1) = 3x_1 + y_1$ and her endowment is $\omega_1 = (0, 10)$. Consumer 2's utility function is $U_2(x_2, y_2) = \ln(x_2) + 2\ln(y_2)$ and his endowment is $\omega_2 = (5, 0)$.

a) Derive the contract curve (the set of Pareto optimal allocations) for this economy (please specify the entire set of points that make up the contract curve).

b) Draw the contract curve in an Edgeworth box. Specify the initial endowment. Carefully draw an indifference curve for each consumer that goes through the point $(x_1, y_1) = (3, 6)$ [this is also the point $(x_2, y_2) = (2, 4)$ for consumer 2].

c) Set up the optimization problem for each consumer and derive the offer curves (demand functions for x_1, y_1, x_2, y_2).

Question 4

Suppose that the coal mining industry in Wonderland is perfectly competitive. Firms operating in the industry are identical, each facing a fixed cost of $1,000 and no marginal cost. The productivity of each firm decreases as more firms operate in the country. A firm's productivity is given by the following function:

$$q = 1{,}000 - n$$

where n denotes the total number of firms. Assume that the price of coal is fixed at $5.

a) How many firms will operate in the industry? Find the total output level of the industry.

b) Now suppose that the coal mining industry is owned by the Wonderland government, which operates as a social planner. How many firms will operate in the industry? Find the total output level of the industry.

c) Now suppose that the Wonderland government cannot directly operate in the coal mining industry. Instead, it seeks to achieve the optimal output level and the optimal number of firms you got in part b) by imposing a lump sum tax ($M), which is a fixed amount of tax regardless of output level. Suppose that the firm cannot pass the tax to consumers by raising the price. How much should the government tax each firm?