
Part A: Answer Question A1 (required) and Question A2 or A3 (choice).

A1 (required): Debt Reduction

Policymakers around the globe are dealing with unprecedented levels of government debt. One option to reduce the national debt is to raise taxes and use the revenue to buy back government bonds from the public. Analyze the effects of this policy change under the following scenarios.

1. Closed Economy: Short vs. Long Run

Consider the following model of aggregate demand with static inflationary expectations and a wealth effect in the goods market. Real wealth held by the public is in the form of money and government bonds. The capital stock and technology are fixed.

$$(1) \quad Y = E(Y-T, R, V, G) \quad \text{where} \quad 0 < E_{Y-T} < 1, E_R < 0, 0 < E_V < 1, E_G = 1 \quad (\text{IS})$$

$$(2) \quad M/P = L(Y, R) \quad \text{where} \quad L_Y > 0, L_R < 0 \quad (\text{LM})$$

The variables are: Y = output, E = aggregate expenditures, T = taxes, R = real/nominal interest rate, G = government purchases of goods and services, $V = (M+B)/P$ = real wealth, B = government bonds held by the public, M = nominal money supply, P = price level, and L = real money demand.

a) For the short run, when wages and prices are fixed:

- i. Show graphically and explain in detail how/why the endogenous variables respond to the combined tax increase/national debt reduction.
- ii. Explain how your answer changes if real money demand is perfectly interest-inelastic.

b) For the long run, when wages and prices are flexible:

- i. Determine how the endogenous variables respond to the combined tax increase/national debt reduction by calculating and signing the relevant derivatives using Cramer's Rule.
- ii. Explain how your answer changes if government bonds are not considered wealth.

2. Small Open Economy: Fixed vs. Flexible Exchange Rates

Consider the basic Mundell-Fleming model with fixed wages/prices, static expectations, and no wealth effects. Assuming perfect capital mobility, use graphical and/or mathematical analysis to explain how/why the small open economy responds to the debt-reducing tax increase under fixed versus flexible exchange rates.

A2 (choice): Optimal Policy

Consider the following version of the Barro-Gordon model, where the policymaker seeks to minimize the social loss function by selecting the optimal inflation rate (π_t) directly:

- (1) $u_t = u_n - \lambda(\pi_t - \pi_t^e)$ with $\lambda > 0$ (Short-run Phillips Curve)
(2) $S_t = (\pi_t - \pi^*)^2 + c(u_t - u^*)^2$ with $\pi^* > 0, u^* < u_n, c > 0$ (Social Loss Function)
(3) $\pi_t^e = E_{t-1} \pi_t$ (Rational Expectations)

The variables are: u = unemployment rate, u_n = natural unemployment rate, u^* = target unemployment rate, π^e = expected inflation rate, π^* = target inflation rate, S = social loss, and E = expectations operator.

- a) Calculate the time-consistent equilibrium inflation rate and the social loss in the discretionary equilibrium.
- b) Explain intuitively, and support your arguments mathematically, whether society will be better or worse off under the following scenarios.
 - i. Unemployment becomes less responsive to unexpected inflation.
 - ii. The policymaker becomes more concerned with unemployment deviations from target.

A3 (choice): Statements

Select any three of the following statements and explain carefully why each is true, false, or uncertain in all its parts. You must use graphical and/or mathematical analysis to support your arguments. Your score depends on the quality and completeness of your explanations.

- a) Given a Solow growth model with labor-augmenting exogenous technology, where a fraction ρ of the population (N) is in the labor force and the population grows at rate $n > 0$, a permanent increase in the labor force participation rate (ρ) has permanent level as well as growth rate effects on output per worker.
- b) Whether the Fisher effect is full or partial depends solely on the interest elasticity of real money demand.
- c) Given rational expectations, pre-announced policies do not affect output in the short run whereas surprise policies may end up destabilizing the economy.
- d) In a stochastic world, whether the optimal policy is a fixed rule or a feedback rule depends on the nature of the uncertainty, the number of policy instruments, and the expectation formation mechanism.

Part B: Answer Both Questions.

B1: Taxes and Dynamic Inefficiency in an OLG Model

Consider an economy consisting of an infinite sequence of two period lived, overlapping generations. N_t agents are born in period t , with $n = 0$. In each period there is a single final good that is produced using a constant returns to scale technology with capital and labor as inputs. Let k_t denote the time t capital-labor ratio, and let $f(k_t)$ denote the intensive production function. Let f have the Cobb-Douglas form $f(k_t) = Ak_t^\alpha$, with $0 < \alpha < 1$. One unit of the final good that is not consumed at t converts into one unit of capital at $t + 1$. Capital does not depreciate after production ($\delta = 0$). Agents have the utility function

$$u(c_{1,t}, c_{2,t+1}) = \frac{c_{1,t}^{1-\theta} - 1}{1-\theta} + (1+\rho)^{-1} \frac{c_{2,t+1}^{1-\theta} - 1}{1-\theta}$$

with $\theta \rightarrow 1$.

For each unit of assets a_t owned by agents in their second period of life, the government gives them a subsidy of σ_{t+1} . In order to balance the budget, the government imposes a tax τ_t on labor income w_t .

- a) Write down the household's maximization problem and derive the equations that characterize the solution. Discuss.
- b) Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form.
- c) What are the equilibrium conditions for this economy? What is the government budget constraint?
- d) Combine your answers to parts a) - c) and derive a *Law of Motion (LoM)* equation that defines a difference equation for the variable k . Get rid of all prices. Looking at it, can we say anything about a steady-state solution? Can you graph the *LoM*?
- e) Is the non-trivial steady-state in the Competitive Equilibrium (CE) Pareto Optimal (PO)? Carefully show and explain why, or why not. Under what conditions will the CE be PO? Can you find an optimal tax, so that the CE is PO?
- f) Do the following comparative dynamics exercise. Initially, the CE economy is with $\sigma = \tau = 0$, and now the government imposes the optimal tax and subsidy rates that you found in part e). Draw (i) *LoM* for both cases, indicating what is different, and (ii) the time paths of the logs of c and k for both cases. Discuss. Compare to question B2.

B2: Taxes and Capital Externalities in an Optimal Growth Model

Consider the model of an economy in competitive equilibrium, where there are capital externalities. There is a representative household and a representative firm. The household's utility functional is

$$U \equiv \int_0^{\infty} u(c_t) e^{-\rho t} dt,$$

with

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta},$$

where $1 > \rho > n = 0$, and $\theta \rightarrow 1$.

The representative firm has a production function $F[K_t, \bar{K}_t, L_t] = K_t^\alpha (\bar{K}_t L_t)^{1-\alpha}$, where \bar{K} is the total quantity of capital in the economy. Normalize $L = 1$, and assume capital does not depreciate after production ($\delta = 0$). For each unit of assets a_t owned by agents, the government gives them a subsidy of σ_t . In order to balance the budget, the government imposes a tax τ_t on labor income w_t .

- a) Write down representative household's maximization problem, solve it, and derive the equations that characterize the solution.
- b) Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form. Derive the equations that characterize the solution.
- c) What are the equilibrium conditions for this economy? What is the government budget constraint?
- d) Combine your answers to parts a) - c) and derive a pair of differential equations for the variables c and k . Can you draw a phase diagram? If yes, draw the phase diagram, carefully identifying (and deriving mathematically) all the important points. Is there a balanced growth path? Show it on the graph, and derive its slope.
- e) What is the growth rate of the economy? What about transitional dynamics?
- f) Is the Competitive Equilibrium (CE) Pareto Optimal (PO)? If yes, why? If not, can we choose optimal subsidy and tax rates, so that the CE becomes PO?
- g) Do the following comparative dynamics exercise. Initially, the CE economy is with $\sigma = \tau > 0$, and now the government removes the optimal subsidy and tax rates that you found in part f). Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of the logs of c and k for both cases. Discuss. Compare to question B1.