**Part A: Answer Question A1 (required) and Question A2 or A3 (choice).**

**A1 (required): Cutting Taxes**

Under the 2017 US Tax Cut and Jobs Act, individual income taxes would be cut by $440 billion over the next ten years. Estimates are that this tax cut, along with other provisions of the bill, would also increase the US national debt over the next ten years. Analyze the effects of the debt-financed tax cut under the following scenarios.

1. *Closed Economy: Short vs. Long Run*

Consider the following model of aggregate demand with static inflationary expectations and a wealth effect in the goods market. Real wealth held by the public is in the form of money and government bonds. Assume that the capital stock and technology are fixed.

(1) $Y = E(Y-T,R,V,G)$ where $0 < E_{Y-T} < 1$, $E_R < 0$, $0 < E_V < 1$, $E_G = 1$ (IS)
(2) $M/P = L(Y,R)$ where $L_Y > 0$, $L_R < 0$ (LM)

The variables are: $Y$ = output/income, $E$ = aggregate expenditures, $T$ = taxes, $R$ = real/nominal interest rate, $G$ = government purchases of goods and services, $V = (M+B)/P$ = real wealth, $B$ = government bonds held by the public, $M$ = nominal money supply, $P$ = price level, and $L$ = real money demand.

a) For the short run, when wages and prices are fixed:

   i. Show graphically and explain in detail how/why the endogenous variables respond to the debt-financed tax cut.

b) For the long run, when wages and prices are flexible:

   i. Determine how the endogenous variables respond to the debt-financed tax cut by calculating, signing, and interpreting the relevant derivatives using Cramer's Rule.

   ii. Explain how your answer changes if government bonds are not considered wealth.

2. *Small Open Economy: Fixed vs. Flexible Exchange Rates*

Consider the basic Mundell-Fleming model with fixed wages/prices, static expectations, and no wealth effects. Assuming perfect capital mobility, use graphical and/or mathematical analysis to explain how/why a small open economy responds to the debt-financed tax cut under fixed versus flexible exchange rates.

**A2 (choice): Economic Growth**

Many emerging economies have seen significant changes in their labor force over the past thirty years. Consider the following growth models to analyze the effects of two such changes.

a) Increase in Labor Force Participation: Consider the following Solow growth model, where total output ($Y$) is a constant-returns-to-scale production function of physical capital ($K$) and effective labor ($AL$).

| | | | |
|---|---|---|---|
| (1) | $Y = K^{a}(AL)^{1-a}$ | where $0 < a < 1$ | (production function) |
| (2) | $dK/dt = sY$ | where $0 < s < 1$ | (capital accumulation) |
| (3) | $dA/dt = g_A A$ | where $g_A > 0$ | (technical progress) |
| (4) | $L = \rho N$ | where $0 < \rho < 1$ | (labor force) |
| (4) | $dN/dt = nN$ | where $n > 0$ | (population growth) |

The other variables are: $A$ = labor-augmenting technology/knowledge, $L$ = labor force, $N$ = total population, $\alpha$ = income share of capital, $s$ = saving rate, $g_A$ = growth rate of technology, $\rho$=labor force participation rate, and $n$ = population growth rate.

i. Characterize the steady-state of this economy by showing the initial equilibrium in a Solow graph and deriving the growth rate of output per worker ($y = Y/L$) on the balanced growth path.

ii. Next, show graphically and explain how/why this economy responds to a permanent (ceteris paribus) increase in $\rho$. Be sure to discuss the effects on the growth rate as well as the level of output per worker over time.

b) Increase in R&D Employment: Consider the following R&D/endogenous growth model without physical capital, where total output ($Y$) and new ideas ($dA/dt$) are produced using rival labor ($L$) and non-rival knowledge ($A$). A fraction $a_L$ of the labor force is employed in the R&D sector. Assume a constant labor force participation rate.

| | | | |
|---|---|---|---|
| (1) | $Y = A(1-a_L)L$ | where $0 < a_L < 1$ | (output production) |
| (2) | $dA/dt = (a_L L)^{\gamma} A^{\theta}$ | where $\gamma > 0, \theta < 1$ | (knowledge production) |
| (3) | $dL/dt = nL$ | where $n > 0$ | (labor accumulation) |

i. Characterize the dynamics of this economy by calculating the growth rate of knowledge in steady-state and determining whether the economy is on a balanced growth path.

ii. Next, show graphically and explain how/why this economy responds to a permanent (ceteris paribus) increase in $a_L$. Be sure to discuss the effects on the growth rate of output per worker over time.

**A3 (choice): Statements**

Select <u>any three</u> of the following statements and explain carefully why each is true, false, or uncertain in all its parts. You must use graphical and/or mathematical analysis to support your arguments. Your score depends on the quality and completeness of your explanations.

a) In the Solow model with human capital, where $H = L \cdot e^{\phi E}$, a permanent (ceteris paribus) increase in years of education ($E$) only has temporary level and growth rate effects on output per worker.

b) Whether or not money is neutral/superneutral under flexible wages and prices depends on the absence or presence of a wealth effect in the goods market.

c) Given rational expectations, pre-announced policies do not affect output in the short run whereas surprise policies may end up destabilizing the economy.

d) According to the Barro-Gordon model, a higher natural unemployment rate and a policymaker with a relatively greater dislike of unemployment will both cause the time-consistent equilibrium inflation rate to increase.

**Part B: Answer Both Questions.**

### B1: Taxes and Dynamic Inefficiency in an OLG Model

Consider an economy consisting of an infinite sequence of two period lived, overlapping generations. $N_t$ agents are born in period $t$, with $n = 0$. In each period there is a single final good that is produced using a constant returns to scale technology with capital and labor as inputs. Let $k_t$ denote the time $t$ capital-labor ratio, and let $f(k_t)$ denote the intensive production function. Let $f$ have the Cobb-Douglas form $f(k_t) = Ak_t^{\alpha}$, with $0 < \alpha < 1$. One unit of the final good that is not consumed at $t$ converts into one unit of capital at $t + 1$. Capital does not depreciate after production ($\delta = 0$). Agents have the utility function

$$u(c_{1,t}, c_{2,t+1}) = \frac{c_{1,t}^{1-\theta} - 1}{1-\theta} + (1+\rho)^{-1} \frac{c_{2,t+1}^{1-\theta} - 1}{1-\theta}$$

with $\theta \to 1$.

For each unit of assets $a_t$ owned by agents in their second period of life, the government gives them a subsidy of $\sigma_{t+1}$. In order to balance the budget, the government imposes a tax $\tau_t$ on labor income $w_t$.

a) Write down the household's maximization problem and derive the equations that characterize the solution. Discuss.

b) Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form.

c) What are the equilibrium conditions for this economy? What is the government budget constraint?

d) Combine your answers to parts a) - c) and derive a *Law of Motion* (*LoM*) equation that defines a difference equation for the variable $k$. Get rid of all prices. Looking at it, can we say anything about a steady-state solution? Can you graph the *LoM?*

e) Is the non-trivial steady-state in the Competitive Equilibrium (CE) Pareto Optimal (PO)? Carefully show and explain why, or why not. Under what conditions will the CE be PO? Can you find an optimal tax, so that the CE is PO?

f) Do the following comparative dynamics exercise. Initially, the CE economy is with $\sigma = \tau = 0$, and now the government imposes the optimal tax and subsidy rates that you found in part e). Draw (i) *LoM* for both cases, indicating what is different, and (ii) the time paths of the logs of $c$ and $k$ for both cases. Discuss. Compare to question B2.

**B2: Taxes and Capital Externalities in an Optimal Growth Model**

Consider the model of an economy in competitive equilibrium, where there are capital externalities. There is a representative household and a representative firm. The household's utility functional is

$$U \equiv \int_0^\infty u(c_t)e^{-\rho t}dt,$$

with

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta},$$

where $1 > \rho > n = 0$, and $\theta \to 1$.

The representative firm has a production function $F\left[K_t, \bar{K}_t, L_t\right] = K_t^\alpha(\bar{K}_t L_t)^{1-\alpha}$, where $\bar{K}$ is the total quantity of capital in the economy. Normalize $L = 1$, and assume capital does not depreciate after production ($\delta = 0$). For each unit of assets $a_t$ owned by agents, the government gives them a subsidy of $\sigma_t$. In order to balance the budget, the government imposes a tax $\tau_t$ on labor income $w_t$.

a) Write down representative household's maximization problem, solve it, and derive the equations that characterize the solution.

b) Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form. Derive the equations that characterize the solution.

c) What are the equilibrium conditions for this economy? What is the government budget constraint?

d) Combine your answers to parts a) - c) and derive a pair of differential equations for the variables $c$ and $k$. Can you draw a phase diagram? If yes, draw the phase diagram, carefully identifying (and deriving mathematically) all the important points. Is there a balanced growth path? Show it on the graph, and derive its slope.

e) What is the growth rate of the economy? What about transitional dynamics?

f) Is the Competitive Equilibrium (CE) Pareto Optimal (PO)? If yes, why? If not, can we choose optimal subsidy and tax rates, so that the CE becomes PO?

g) Do the following comparative dynamics exercise. Initially, the CE economy is with $\sigma = \tau > 0$, and now the government removes the optimal subsidy and tax rates that you found in part f). Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of the logs of $c$ and $k$ for both cases. Discuss. Compare to question B1.