
Part A: Answer Question A1 (required) and Question A2 or A3 (choice).

A1 (required): Short-Run Stabilization Policy and Economic Shocks

In response to economic shocks, policymakers often try to stabilize output in the short run. The appropriate stabilization policy depends on the type of shock, the degree of openness or capital mobility, and many other factors. Using the short-run/Keynesian IS-LM model (i.e., wages/prices are sticky or fixed; the capital stock and technology are constant), analyze the effects of random IS shocks and determine the best policy response under the following scenarios.

1. *Closed Economy*: Consider the following stochastic IS-LM model with static inflationary expectations and random IS shocks.

(1) $Y = E(Y\text{-}T,R,G) + \varepsilon$ where $0 < E_{Y-T} < 1$, $E_R < 0$, $E_G = 1$, $\varepsilon \sim N(0,\sigma^2)$ (IS)

(2) $M/P = L(Y,R)$ where $L_Y > 0$, $L_R < 0$ (LM)

The variables are: Y = real output, E = aggregate expenditures, T = taxes, $R = r$ = interest rate, G = government purchases, M = nominal money supply, P = (fixed) price level, L = real money demand, and ε = random expenditure shock in the goods market.

a) Illustrate graphically and calculate/sign the relevant partial derivatives to show how this closed economy responds to a negative expenditure shock in the goods market ($\varepsilon < 0$).

b) In the face of random expenditure shocks in the goods market, what is the best monetary policy to stabilize output in the short run? Discuss.

2. *Small Open Economy*: Consider a stochastic Mundell-Fleming model with perfect capital mobility, static inflationary expectations, static exchange rate expectations, and random domestic IS shocks.

a) Using graphical analysis, explain carefully how/why this small open economy responds to a negative expenditure shock in the domestic goods market ($\varepsilon < 0$) under both fixed and flexible exchange rates.

b) In the face of random expenditure shocks in the domestic goods market, what is the best exchange rate policy to stabilize domestic output in the short run? Discuss.

A2 (choice): Physical Capital and the Long Run

Physical capital plays an important role in long-run economic analysis. You are asked to analyze the effects of a ceteris paribus change in a country's capital stock in the context of the following models.

1. <u>*Classical/Long-Run Model*</u>: Consider the following aggregate supply/demand model where wages and prices are flexible, inflationary expectations are static, and technology is fixed.

(1)	$W/P = F_N(N,K)$	where $F_{NN} < 0$, $F_{NK} > 0$	(labor demand)
(2)	$N = NS(W/P)$	where $NS_{W/P} > 0$	(labor market equilibrium)
(3)	$Y = F(N,K)$	where $F_N > 0$, $F_K > 0$	(production function)
(4)	$Y = E(Y\text{-}T,R,G)$	where $0 < E_{Y-T} < 1$, $E_R < 0$, $E_G = 1$	(IS)
(5)	$M/P = L(Y,R)$	where $L_Y > 0$, $L_R < 0$	(LM)

The variables are: W/P = real wage, N = employment/labor, K = capital stock, Y = real output, E = aggregate expenditures, T = taxes, $R = r$ = interest rate, G = government purchases, M = nominal money supply, P = price level, and L = real money demand.

Suppose a natural disaster destroys part of the country's capital stock.

a) Illustrate graphically how the endogenous variables are affected.

b) Explain carefully how/why the endogenous variables are affected.

2. <u>*Solow Growth Model*</u>: Consider the following Solow model with a Cobb-Douglas production function.

(1)	$Y = K^a (AL)^{1-a}$	where $0 < a < 1$	(production function)
(2)	$dK/dt = sY - \delta K$	where $0 < s < 1$, $0 < \delta < 1$	(capital accumulation)
(3)	$dL/dt = nL$	where $n > 0$	(labor accumulation)
(4)	$dA/dt = gA$	where $g > 0$	(technical progress)

The variables are: Y = total output, A = labor-augmenting technology, K = capital stock, L = labor force, a = income share of capital, n = population growth rate, s = saving rate, δ = depreciation rate, g = rate of technical progress.

a) Calculate the level of output per worker and the growth rate of output per worker in steady-state.

b) Now suppose a natural disaster destroys part of the country's capital stock.

   (i) Show graphically and explain carefully how/why output *per effective worker* is affected over time.

   (ii) Using time series graphs and verbal explanations, discuss what happens to the level of output *per worker* and the growth rate of output *per worker* over time.

A3 (choice): Statements

Select any three of the following statements and explain carefully why each is true, false, or uncertain in all its parts. You must use graphical and/or mathematical analysis to support your arguments. Your score depends on the quality and completeness of your explanations.

a) In a closed economy, whether the Fisher effect is full or partial depends entirely on the degree of wage and price flexibility.

b) In a stochastic world, whether the optimal policy is a fixed rule or a feedback rule depends entirely on the nature of the uncertainty.

c) According to the Solow growth model, output per worker tends to be higher in countries with high saving rates, high skill ratios, high levels of technology, and high population growth rates, all else equal.

d) In the R&D growth model, a ceteris paribus increase in the share of labor employed in the R&D sector will permanently raise the growth rate of output per worker.

Part B: Answer Both Questions.

B1. Taxation in the Ramsey Model

Consider the Ramsey model of an economy in competitive equilibrium. There is a representative household and a representative firm. The household's utility functional is

$$U \equiv \int_0^\infty \ln c_t e^{-\rho t} dt,$$

where there is no population growth, and $\rho > 0$.

The representative firm has a constant returns to scale per worker production function $f(k_t) = Ak_t^\alpha$. For simplicity, assume capital does not depreciate after production ($\delta = 0$). At every point in time, the household must pay a fraction τ of its income to the government.

Find the competitive equilibrium of this economy, using the following steps.

a) Write down representative household's maximization problem, solve it, and derive the 4 equations that characterize the solution. Explain in words, intuitively, what the Hamiltonian function means, and what the 4 equations represent. Does τ show up here? Explain why or why not?

b) Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form. Derive the 2 equations that characterize the solution. Does τ show up here? Explain why or why not?

c) What are the 3 equilibrium conditions for this economy? Does τ show up here? Explain why or why not?

d) Combine your answers to parts a) - c) and derive a pair of differential equations for the variables c and k.

e) Draw the phase diagram, carefully identifying (and deriving mathematically) all the important points.

f) Do the following comparative dynamics exercise: $\tau' > \tau$. Explain why there is no need to assume that the substitution effect dominates the income effect. As usual, the baseline economy starts in its steady state at time $t = 0$. The modified economy starts at time $t = 0$ (with the same amount of capital as the baseline economy. Remember that capital is the state variable!). Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of c and k for both cases. In interpreting your results, notice that we completely ignored what the government may do with the tax revenues. For simplicity, we *de facto* assumed that the government wasted the taxes, "dumped them into the ocean", or "opened a Swiss bank account" with them. Thus, there is no positive effect of government spending countering the income tax.

B2. Financial Intermediation

Consider the Diamond-Dybvig model with two assets. There are three periods: $t = 0, 1, 2$. Agents are ex-ante identical. They are endowed one unit of a single good at $t = 0$, and nothing at $t = 1, 2$. At the beginning of $t = 1$, a fraction ϕ_1 of agents learn that they prefer to consume only at $t = 1$, while a fraction ϕ_2 of agents prefers to consume only at $t = 2$. There is a linear production technology whereby one unit of the good invested in period 0 yields $R>1$ units of the good at time 2. This technology is illiquid, in the sense that an investment that is interrupted in period 1 generates $r<1$ units of consumption. In addition, there is a liquid storage technology, whose return is equal to 1 in both periods. Agents are expected utility maximizers, their preferences are given by

$$U = \phi_1 \frac{c_1^{1-\theta} - 1}{1-\theta} + \rho\phi_2 \frac{c_2^{1-\theta} - 1}{1-\theta} \tag{1}$$

where $\rho < 1$ is a discount factor, and $\theta > 0$.

a) Write down the problem of an agent in autarky, the FOC, and the optimal consumption vector (c_1, c_2).

b) Now suppose that in period 1, after agents learn their idiosyncratic consumption preference shock and before they consume, a financial market opens where agents can trade claims for the returns on the illiquid production technology. Let p be the price of a bond that yields one unit of the illiquid production technology at $t = 2$. Write down the problem of an agent in this setting. What will the equilibrium price of a bond be in this case (and why)? What is the consumption vector (c_1, c_2)? Discuss.

c) Now, instead of a financial market, suppose agents form coalitions, which they call banks, and pool their resources. Write down the problem of the bank, the FOC, and the optimal consumption vector. Briefly compare with a) and b).

d) Under what conditions can multiple equilibria exist? Define narrow banking in this context. Is it preferred to autarky? Discuss. What about suspension of convertibility?