Ph.D. Core Exam -- Macroeconomics **15 August 2016 -- 8:00 am to 3:00 pm**

Part A: Answer question A1 (required), plus either question A2 or A3.

A1 (required): Macroeconomic Effects of "Brexit"

In the wake of the recent British decision to leave the European Union ("Brexit"), economists around the globe are debating the potential macroeconomic effects for Great Britain (aka the UK) and the EU. You are asked to contribute to this debate, using your knowledge of models of macroeconomic fluctuations.

1. First consider the case of the UK. For the short run, suppose the British economy can be described by a Mundell-Fleming (aka open-economy IS-LM) model with flexible exchange rates, perfect capital mobility, static inflationary expectations, and static exchange rate expectations.

   a) Assume that Brexit generates negative random shocks in the UK goods and money markets. (i) Using graphical and verbal analysis, explain how/why the British economy responds to these shocks in the short run. (ii) Discuss what the British government can/should do to stabilize output in the short run.

   b) Discuss at least two other economic channels/parameters/variables (apart from those mentioned in part a) through which Brexit is likely to affect the British economy in the short to medium run.

2. Next consider the case of the EU. Suppose the post-Brexit EU economy can be described by the following model of aggregate demand with autonomous net exports (*NX*), static inflationary expectations ($d\pi^e = \pi^e = 0$), fixed capital stock, and fixed technology.

   (1) $Y = E(Y\text{-}T, R, G, NX)$ where $0 < E_{Y-T} < 1$, $E_R < 0$, $E_G = 1$, $E_{NX} = 1$ (IS)

   (2) $M/P = L(Y,R)$ where $L_Y > 0$, $L_R < 0$ (LM)

The other variables are: Y = real output/income, E = aggregate expenditures, T = taxes, R = nominal/real interest rate, G = government purchases of goods and services, M = nominal money supply, P = price level, L = real money demand.

Assume that Brexit results in an exogenous drop in EU net exports. Analyze the effects of this (ceteris paribus) change on the EU economy for the following time horizons.

   a) <u>Short run with fixed wages/prices</u>: Show graphically and explain in detail how/why the relevant endogenous variables respond in the short run.

   b) <u>Long run with flexible wages/prices</u>: Determine how the relevant endogenous variables respond in the long run by calculating and signing the corresponding derivatives.

A2: European Economic Growth and Migration

Over the past two years, European countries have experienced an unprecedented influx of migrants from the Middle East and Northern Africa. Assuming that this influx creates a one-time increase in the European labor force (L) without raising the population growth rate (n), analyze the impact on the European economy in context of different growth models.

1. Consider a basic Solow growth model, where total output (Y) is a constant-returns-to-scale production function of physical capital (K) and effective labor (AL).

(1)	$Y = K^{\alpha}(AL)^{1-\alpha}$	where $0 < \alpha < 1$	(production function)
(2)	$dK/dt = sY$	where $0 < s < 1$	(capital accumulation)
(3)	$dL/dt = nL$	where $n > 0$	(labor accumulation)
(4)	$dA/dt = g_A A$	where $g_A > 0$	(technical progress)

The other variables are: A = labor-augmenting technology/knowledge, α = income share of physical capital, s = saving rate, g_A = growth rate of technology.

a) Characterize the initial equilibrium for this economy by (i) showing the steady-state equilibrium in a Solow graph, (ii) calculating output per worker ($y = Y/L$) in steady-state, and (iii) deriving the growth rate of output per worker on the balanced growth path.

b) Illustrate graphically and explain how/why both the level and the growth rate of output per worker respond over time to the one-time (ceteris paribus) increase in the European labor force.

2. Now consider a simple R&D/endogenous growth model without physical capital, where a fraction a_L of the labor force is employed in the R&D sector.

(1)	$Y = A(1-a_L)L$	where $0 < a_L < 1$	(output production)
(2)	$dA/dt = (a_L L)^{\gamma} A^{\theta}$	where $\gamma > 0, \theta < 1$	(knowledge production)
(3)	$dL/dt = nL$	where $n > 0$	(labor accumulation)

a) Characterize the dynamics this economy by (i) calculating the growth rate of knowledge in steady-state and (ii) determining whether the economy is on a balanced growth path.

b) Explain how/why both the level and the growth rate of output per worker respond over time to the one-time (ceteris paribus) increase in the European labor force, comparing and contrasting the cases where (i) migrants include a fraction a_L of scientists/engineers employed in Europe's R&D sector and (ii) migrants include no scientists/engineers, only workers employed in Europe's output sector.

A3: Statements

Select <u>any three</u> of the following statements and explain carefully why each is true, false, or uncertain in all its parts. You must use graphical and/or mathematical analysis to support your arguments. Your score depends on the quality and completeness of your explanations.

1. In the Solow growth model with human capital formation given by $H = L \cdot e^{\psi E}$, a ceteris paribus decrease in education (E) has no permanent effects on the steady-state level of output per worker and the economy's balanced growth path.

2. In the presence of wealth effects, money ceases to be neutral and superneutral.

3. Given rational expectations, neither expected nor unexpected monetary policy has real effects in the short run.

4. In a stochastic world, whether the optimal policy is a fixed rule or a feedback rule depends on the nature of the uncertainty, the number of policy instruments, and the expectation formation mechanism.

Part B: Answer Both Questions

B1: Fully Funded Social Security

Consider an economy consisting of an infinite sequence of two period lived, overlapping generations. N_t agents are born in period t, with $N_{t+1} = (1+n)N_t$. In each period there is a single final good that is produced using a constant returns to scale technology with capital and labor as inputs. Let k_t denote the time t capital-labor ratio, and let $f(k_t)$ denote the intensive production function. Let f have the Cobb-Douglas form $f(k_t) = Ak_t^{\alpha}$, with $0 < \alpha < 1$. One unit of the final good that is not consumed at t converts into one unit of capital at $t+1$. Capital depreciates after production, with $\delta \in (0,1)$. Agents have the utility function

$$u(c_{1,t}, c_{2,t+1}) = \frac{c_{1,t}^{1-\theta} - 1}{1-\theta} + (1+\rho)^{-1} \frac{c_{2,t+1}^{1-\theta} - 1}{1-\theta}$$

with $\theta > 0$.

Suppose that, besides saving in assets (a_t), young agents born in period t are forced by the government to contribute some amount to social security (label this variable d_t). These funds are invested in capital, and agents receive the returns, given by $(1+r_{t+1})d_t$, when they are old.

a) Write down the household's maximization problem and derive the equations that characterize the solution. Discuss.

b) Write down the firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form.

c) What are the equilibrium conditions for this economy? Pay particular attention to the savings=investments equilibrium condition.

d) Combine your answers to parts (a) - (c) and derive a *Law of Motion* (*LoM*) equation that defines a difference equation for the variable k. Looking at it, can we say anything about a steady-state solution? Can you graph the *LoM?*

e) Is the non-trivial steady-state in the Competitive Equilibrium (CE) Pareto Optimal (PO)? Carefully show and explain why, or why not. If not, how could you modify this government program to make it PO?

B2: Growth and Infrastructure

Consider the Ramsey model of an economy in competitive equilibrium. There is a representative household and a representative firm. The household's utility functional is

$$U \equiv \int_0^\infty u(c_t) e^{-\rho t} dt,$$

with

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta},$$

where $1 > \rho > n = 0$, and $\theta > 0$.

The representative firm has a production function

$$F[K_t, G_t, L_t] = AK_t^\alpha (G_t L_t)^{1-\alpha},$$

where G is the total quantity of infrastructure provided by the government in this economy. Further assume infrastructure grows at a constant rate g. That is,

$$\dot{G}_t = gG_t$$

For simplicity, normalize $L = 1$, and assume capital does not depreciate after production ($\delta = 0$). Find the competitive equilibrium of this economy, using the following steps.

a) Write down the representative household's maximization problem, solve it, and derive the 4 equations that characterize the solution.

b) Write down the firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form. Derive the 2 equations that characterize the solution.

c) What are the equilibrium conditions for this economy?

d) Combine your answers to parts a) - c) and derive a pair of differential equations for the variables c and k. Can you draw a phase diagram? If so, carefully identify (and derive mathematically) all the important points. If you can't draw a phase diagram, can you transform the differential equations in order to be able to draw a phase diagram? Is there a balanced growth path? What is its slope? What is the growth rate of the economy?

e) Is the Competitive Equilibrium Pareto Optimal?

f) Do the following comparative dynamics exercise: $g' > g$. Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of the logs of c and k for both cases. Discuss.