Ph.D. MICROECONOMICS CORE EXAM
August 2018

This exam is designed to test your broad knowledge of microeconomics. There are three sections: one required and two choice sections. You must complete both problems in the required section and one choice problem in each of the two choice sections, giving you a total of four problems to complete during the allotted time. The required problems are in section A and the choice problems are in sections B and C. If you should answer more than one choice question in a section, only the first will be considered.

IMPORTANT. You are expected to adhere to the following guidelines in completing the exam for your answer to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number all pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number.**
- Provide clear, concise discussion to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Explain your steps in any mathematical derivations. Simplify your final answers completely.
- When you turn in your exam answers double check to make sure you have included all the pages to each question number, and in order. The pages you submit as your answer are the only ones that will be considered.
- To simplify copying, please leave 1 inch borders.

<u>PART A: REQUIRED QUESTIONS</u>

Both problems in Part A (A1 and A2) are <u>required</u>. Answer all parts of all questions.

QUESTION A1

Suppose Amy consumes two goods. Her utility function is $u(x_1, x_2) = (x_1 - a_1)^{0.5}(x_2 - a_2)^{0.5}$. Amy's income is w and the prices for the two goods are p_1 and p_2, respectively.

a) Derive Amy's indirect utility function.

b) Derive Amy's Hicksian demand functions. Verify that Shephard's Lemma is satisfied.

c) For the remaining of the question, assume $a_1 = a_2 = 0$. Suppose that there is a 20% chance that p_2 will increase to $4p_2$. Amy has the option to purchase insurance at a premium C. If she purchases the insurance and the price increase occurs, she will receive a fixed amount, B, from the insurance company. If she purchases the insurance and the price increase does not occur, she will not receive any payments from the insurance company. Find the highest C that Amy would be willing to pay for the insurance.

d) Now suppose that instead of paying a fixed amount B, the insurance company will reimburse 75% of the amount that Amy pays for good 2 if the price increase occurs. Find the highest C that Amy would be willing to pay for the insurance.

QUESTION A2

Consider a 2×2 exchange economy. Consumer 1's utility function is $U_1(x_1, y_1) = x_1 + 4\ln(y_1)$ and her endowment is $\omega_1 = (2, 8)$. Consumer 2's utility function is $U_2(x_2, y_2) = x_2 + y_2$ and his endowment is $\omega_2 = (8, 2)$.

a) Derive the contract curve, the set of Pareto optimal allocations, for this economy. [This can be done either with an analytical solution or a carefully drawn Edgeworth box (or both).]

b) Set up the optimization problem for each consumer and derive the offer curves (demand functions for x_1, y_1, x_2, y_2).

c) Find the competitive equilibrium for this economy. This will be a price vector $p^\star$ together with an allocation $(x^\star, y^\star)$. Confirm that this outcome is Pareto optimal.

d) Now suppose there is a new allocation: $\omega_1 = (8, 8)$, $\omega_2 = (2, 2)$. Find the competitive equilibrium with this new allocation.

PART B: CHOICE QUESTIONS

Answer all parts of either question B1 or B2. If you complete more than one problem, only B1 will be considered.

QUESTION B1

For the true/false/uncertain questions below, please state whether the question is true, false, or uncertain and then you are required to provide a detailed mathematical and written justification. Credit depends only on your explanation.

a) True/False/Uncertain: For any given constant returns to scale (CRS) production function of the form $y = f(x_1, x_2)$, if the average product of x_1 is rising, then the marginal product of x_2 is positive.

b) Assume a production function for a firm is given by $y = \left(\sum_{i=1}^{n} \alpha_i x_i^{\rho}\right)^{1/\rho}$ where $\sum_{i=1}^{n} \alpha_i = 1$ and $\rho \neq 0$ and $\rho < 1$. Verify that the elasticity of substitution between i and j is equal to $1/(1-\rho)$ for all $i \neq j$.

c) Suppose a consumer has indifference curves that are all parallel, negatively sloped straight lines with preferences increasing away from the origin. (i) Prove (or rigorously demonstrate) that these preferences are convex. (ii) Then, prove (or rigorously demonstrate) that these preferences are not strictly convex.

d) Provide an example mathematical utility function, example mathematical budget constraint, and hypothetical price change where EV > CV. For credit, use must mathematically calculate CV and EV for the utility function, budget constraint, and hypothetical price change you consider. State any assumptions you make.

QUESTION B2

There are 600 people distributed uniformly over a one-mile beach. Each person values an ice-cream at $1. You are the owner of an ice-cream shop that currently locates at one end of the beach. The transportation cost (for consumers) is $1 per mile each way.

a) Suppose you can produce ice-creams at no cost. Set up the profit-maximization problem and find the profit-maximizing price.

b) Now assume that the cost function is C(Q)=20+1/3Q. What is the profit-maximizing price that you should charge?

c) Now assume again that you can produce ice-creams at no cost. Suppose that you can choose to relocate at no cost. What is the profit-maximizing price that you should charge?

d) Now assume that you can produce ice-creams at no cost. However, if you choose to relocate, the cost of relocation is $360 per mile. Find the optimal location and the profit-maximizing price.

PART C: CHOICE QUESTIONS

Answer all parts of either question C1 or C2. If you complete more than one problem, only C1 will be considered.

QUESTION C1
Joe has been inspired by the recently completed world Tour de France and is considering changing careers and becoming a professional bicycle rider. If he chooses this career path, there is a 1% chance that he will be very successful and become the team leader, where he'd see a payoff of $500. But there is a 99% chance that he will not become a team leader and in a support role, his payoff would be $5. If he stays in his current career, his payoff is $6. Joe's expected utility is derived from his payoff, *m* - described by $U = 1 - m^{-1}$.

a) Should Joe become a professional bike rider? Be explicit and explain your answer.

b) Utilizing the concept of risk preferences, what is the minimum change in the probability of success as a bike rider, or the minimum change necessary in the payoff of becoming a successful bike rider, needed in order to have Joe switch from his choice in a)? Provide numerical answers for both the probability of success and for the change in the payoff. Also, provide a graphical and verbal explanation of these results.

c) Suppose Joe can hire a bike guru who guarantees with certainty that he, after observing Joe's riding style, will be able to tell Joe whether or not Joe will be very successful as a professional bike rider and become a team leader. Joe thinks this may sound too good to be true, but the bike guru persists and offers Joe a binding contract to offset any loses that Joe would incur if his assessment were incorrect. What is the maximum amount that Joe should pay for the information that can be provided by the bike guru? Again, show your work and explain your answer.

d) As an alternative to the bike guru, Joe can also opt to take training courses that will increase his probability of being very successful as a bike rider. The longer he takes courses, the higher his probability of being very successful, but the higher the cost, *c*. Given this, find Joe's optimal level of training courses. Explain your approach and graph and explain your result.

QUESTION C2

Firms 1 and 2 compete as quantity-setting duopolists. Suppose that each firm is restricted to producing only integer values and may choose to set its quantity, q_i, equal to 1, 2, 3, or 4 units. Each firm has production costs given by $C(q_i) = 2q_i$. Assume the market inverse demand is given by $P(Q) = 8 - Q$, where P is the market price and $Q = q_1 + q_2$.

a) Assume the two firms choose quantity simultaneously. Write the game in normal form (*i.e.*, define a 4×4 payoff matrix). Find all pure-strategy Nash equilibrium for this game.

b) Does a weakly dominant-strategy equilibrium exist for this game? Explain. If the firms can only choose q_i equal to 1, 2, or 3 units (eliminate the fourth row and fourth column), does a weakly dominant-strategy equilibrium exist? Explain.

c) Now suppose the firms play the stage game from part (a) in an infinitely repeated game. Assume the discount factor between each period is δ, with $0 < \delta < 1$. For what range of discount factors is it possible to support the following strategy as a subgame-perfect equilibrium: in even time periods ($t = 0, 2, 4, 6, \ldots$) $q_1 = 2, q_2 = 1$; in odd time periods ($t = 1, 3, 5, 7$) $q_1 = 1$, $q_2 = 2$; and if some firm deviates from this strategy then both firms revert to producing 2 units every period thereafter. Potentially helpful math notes:

$$\sum_{t=0}^{\infty} x\delta^t = \frac{x}{1-\delta} \text{ and } \sum_{t=0}^{\infty} x\delta^{2t} = \frac{x}{1-\delta^2}.$$

d) Now assume that the firms can choose any positive quantity (they are not restricted to choosing integer values) and will play only once. Assume the market inverse demand curve is given by $P(Q) = A - Q$, where A can take on the values 14 or 26. Assume the probability of each possible value is $1/2$. Suppose firm 1 has done market research and knows the value of A but that firm 2 does not. Each firm still has production costs given by $C(q_i) = 2q_i$. Solve for the Bayesian Nash equilibrium in this game when firms choose strategy simultaneously.