Ph.D. MICROECONOMICS CORE EXAM
January 2019

This exam is designed to test your broad knowledge of microeconomics. There are three sections: one required and two choice sections. You must complete both problems in the required section and one choice problem in each of the two choice sections, giving you a total of four problems to complete during the allotted time. The required problems are in section A and the choice problems are in sections B and C. If you should answer more than one choice question in a section, only the first will be considered.

IMPORTANT. You are expected to adhere to the following guidelines in completing the exam for your answer to be considered complete. Incomplete answers will be evaluated accordingly.

- Write legibly. **Number all pages and organize your answers to questions in the same order as they were given to you in the exam. Begin your answer to each question on a new page and identify the question number.**
- Provide clear, concise discussion to your answers.
- Explicitly state all assumptions you make in a problem. Graders will not take unstated assumptions for granted. Do not make so many assumptions as to trivialize or assume the problem away.
- Define any notation you use in a problem and label all graphs completely.
- Explain your steps in any mathematical derivations. Simplify your final answers completely.
- When you turn in your exam answers double check to make sure you have included all the pages to each question number, and in order. The pages you submit as your answer are the only ones that will be considered.
- To simplify copying, please leave 1 inch borders.

<u>PART A: REQUIRED QUESTIONS</u>

Both problems in Part A (A1 and A2) are <u>required</u>. Answer all parts of all questions.

QUESTION A1

Suppose Ada lives for two periods. She is endowed with A units of corn that she can consume or plant in period 1. If she plants s units of corn in period 1, the yield in period 2 will be determined by $\mathrm{f(s) = ts}$ where $\mathrm{t} > 1$. Her utility function is $\mathrm{u}(c_1, c_2) = \ln c_1 + \ln c_2$ where c_1 is her consumption in period 1 and c_2 is her consumption in period 2. Any corn not consumed or planted in period 1 will rot in period 2.

a) Solve for Ada's optimal consumption levels in each period.

b) How would Ada's "indirect utility" (the maximum utility she can achieve) change if t increases? Verify that the optimal consumption in period 2 satisfies a form of Roy's identity (regarding the partial derivatives of the indirect utility with respect to t).

c) Now suppose that the yield in period 2 is affected by the weather in period 2. If the weather is good, the yield in period 2 will be $\mathrm{f}^g(\mathrm{s}) = t_1\mathrm{s}$, where $t_1 > 1$. If the weather is bad, the yield in period 2 will be $\mathrm{f}^b(\mathrm{s}) = t_2\mathrm{s}$, where $t_2 < 1$. The probability of good weather is α and the probability of bad weather is 1- α, $0 < \alpha < 1$. Ada cannot foresee the weather when she chooses how much to plant in period 1. Solve for Ada's optimal consumption levels in each period.

d) Now suppose that $\mathrm{t} = 2$ regardless of the weather in period 2. However, Ada's crop suffered from seedcorn maggot. As a result, t decreased to 1.5. How much corn should be added to Ada's initial endowment so that she can achieve the same utility level as when $\mathrm{t} = 2$?

QUESTION A2

Consider a 2×2 exchange economy. Consumer 1's utility function is $U_1(x_1, y_1) = x_1 + 2y_1$ and her endowment is $\omega_1 = (2, 6)$. Consumer 2's utility function is $U_2(x_2, y_2) = x_2 y_2$ and his endowment is $\omega_2 = (2, 2)$.

a) Derive the contract curve, the set of Pareto optimal allocations, for this economy. Draw an Edgeworth box showing the initial allocation, the contract curve, and a few (rough) representative indifference curves for each consumer.

b) Set up the optimization problem for each consumer and derive the offer curves (demand functions for x_1, y_1, x_2, y_2).

c) Find the competitive equilibrium for this economy. This will be a price vector $p^\star$ together with an allocation $(x^\star, y^\star)$. Confirm that this outcome is Pareto optimal.

d) Now suppose there is a new allocation: $\omega_1 = (2, 0)$, $\omega_2 = (2, 8)$. Find the competitive equilibrium with this new allocation (price vector and allocation).

PART B: CHOICE QUESTIONS

Answer all parts of either question B1 or B2. If you complete more than one problem, only B1 will be considered.

QUESTION B1

Suppose Greg's utility function is $U(x, y) = x^{0.5} + y^{0.5}$. Assume strictly positive prices p_x and p_y and strictly positive income M.

a) Find Greg's Marshallian demand for goods x and y.

b) Part (b) has two sub-parts:
   i. Construct the Hessian. Using the Hessian, explain what the utility function and the related level curves look like. Prove or rigorously demonstrate in sufficient mathematical detail that the utility function is convex towards the origin.
   ii. Draw some representative examples of the level curves.

c) What axiom of consumer preference does this utility function violate? Prove it or rigorously demonstrate in sufficient mathematical detail.

d) Now, add a full set of non-negativity (inequality) constraints to the utility maximization problem for both x and y. Additionally, assume now that M is greater than or equal to total expenditures on goods x and y (i.e., do not assume that all income is spent on the two consumption goods).
   i. Write out the Kuhn-Tucker Lagrangian and solve for the Kuhn-Tucker first-order conditions for a maximum. Don't forget the complementary slackness condition(s).
   ii. Using the Kuhn-Tucker FOC's, solve for Greg's Marshallian demand. You must check for all corner solutions as part of your answer. You must also check the cases where $p_x x + p_y y < M$ and $p_x x + p_y y = M$.
   iii. Do you find a corner solution Marshallian demand? Explain.

QUESTION B2

Consider a monopolist producing one good that can be sold in Albuquerque and Denver. The market demand in Denver is $q_1 = 1000 - 10\mathrm{p}_1$ and the market demand in Albuquerque is $q_2 = 400 - 5\mathrm{p}_2$. The cost of production is $\mathrm{c}(\mathrm{Q}) = 4Q$, where $\mathrm{Q} = q_1 + q_2$. Assume that consumers in Albuquerque cannot access the market in Denver and consumers in Denver cannot access the market in Albuquerque. The firm currently locates in Albuquerque. It would cost $\$\mathrm{t}$ per unit to ship the good to Denver.

<u>In parts a) and b), assume that $\mathrm{t} = 0$.</u>

a) Suppose that the monopolist can set different prices in Denver and Albuquerque. What prices would the monopolist set?

b) Suppose that the monopolist can only set one price in both cities. What price would the monopolist set?

<u>In parts c) and d), assume that $\mathrm{t} = 2$.</u>

c) Now suppose that the monopolist can set different prices in Denver and Albuquerque. The firm can move to Denver at a cost M. If the firm moves to Denver, there will be no shipping cost for goods sold in Denver. However, selling the good in Albuquerque would cost $\$\mathrm{t}$ per unit to ship. Find the firm's maximum willingness to pay for the move.

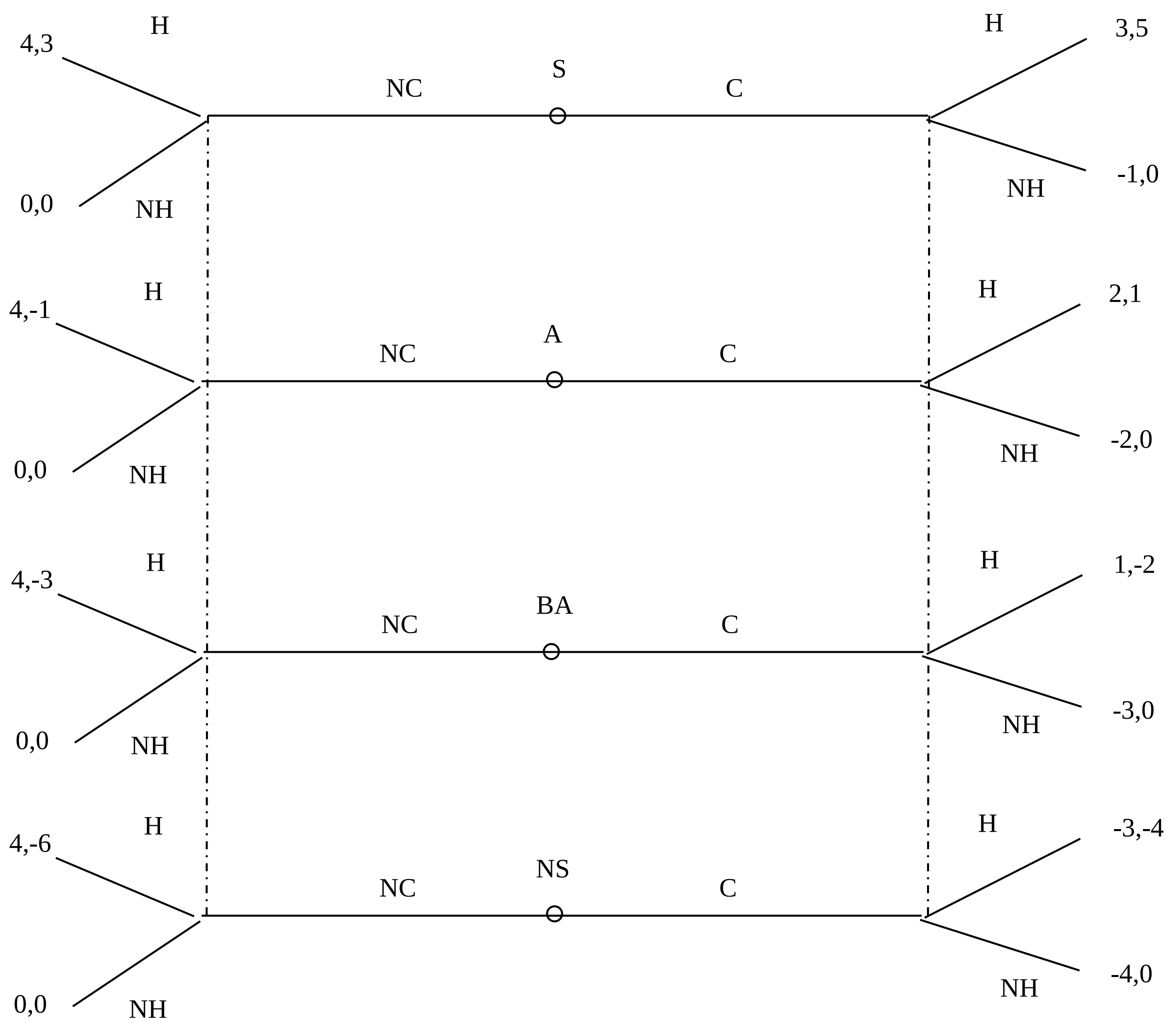
d) How would your result in part c) differ if the monopolist cannot set different prices in Denver and Albuquerque?

PART C: CHOICE QUESTIONS

Answer all parts of either question C1 <u>or</u> C2. If you complete more than one problem, only C1 will be considered.

QUESTION C1

Consider the following signaling game. In this game the sender can be one of four potential types: smart (S), average (A), below average (BA), and not smart (NS). Assume that these types are determined by nature (and by the local k-12 education system) and only observed by the sender (not the receiver). The sender observes his/her type and then chooses a message $M = \{NC, C\}$, where NC indicates not going to college, and C indicates going to college. Once the receiver observes this message, she chooses to either hire (H) or not hire (NH) the sender. (The payoffs are listed with the payoff for the receiver on the left and the payoff for the receiver on the right) **<u>For all answers describe your notation.</u>**



a) Describe the payoffs for this game. Does college improve productivity? Is college costly? Do these payoffs make sense? Next, assume that p(S)=.5, p(A)=0, p(BA) =0 and p(NS)=.5.

What are the pure-strategy NE for this game? Discuss.

b) Assume that p(S)=.2, p(AA)=.0, p(BA) =.6 and p(NS)=.2. What are the pure-strategy NE for this game? Discuss.

c) Now assume that p(S)=.2, p(AA)=.6, p(BA) =0 and p(NS)=.2. What are the pure-strategy NE for this game? Discuss.

d) What is the difference between the equilibria derived in b and the equilibria designed in c? Can you think of any policy implications of these findings in terms of education policy in the US (particularly poor/low-income communities)?

QUESTION C2

Two firms produce a homogeneous good with a market inverse demand for the good of $P(Q) = 6 - Q$, where $Q = q_1 + q_2$, and q_i is the output for firm i. Each firm has costs of $C(q_i) = 2q_i$.

a) Assume that both firms choose production quantities simultaneously. Solve for the Nash equilibrium for this game, and the equilibrium profits for each firm.

b) Now assume that firm 1 chooses their quantity before firm 2. Solve for the subgame perfect Nash equilibrium in this game, and the equilibrium profits for each firm.

c) Now suppose that there are two stages to the game. In the first stage, each firm simultaneously chooses whether to commit to a quantity choice. Then, in the second stage, the firms play the following game. If both firms have committed to a quantity choice, the firms produce this quantity and price is determined by the inverse demand function. If neither firm commits, the standard Cournot game is played. If one firm commits to a quantity in stage one while the other does not, then the committed firm produces the quantity it has committed to and the rival firm chooses its quantity in response (like the game played in part b). To simplify the analysis, assume that the firms have only three possible moves in the first stage: they may commit to a quantity of 1, a quantity of 2, or they may choose not to commit. Find all pure strategy subgame perfect Nash equilibria for the two-stage game.

d) Finally, suppose the firms will play an infinitely repeated simultaneous quantity choice game. What range of discount factors can support the strategy of both firms choosing a quantity of 1 in every period, where if either firm deviates the other will play the Cournot strategy for every strategy thereafter? Potentially helpful equations: $\sum_{t=0}^{\infty} \delta = \frac{1}{1-\delta}$, $\sum_{t=1}^{\delta} \delta = \frac{\delta}{1-\delta}$.