1) Consumer Theory I

An economy consists of two consumers with labels $i = 1, 2$. They exchange two goods $j = 1, 2$. Suppose there is a fixed total endowment e_j of each good to be distributed between the two consumers. Let c_j^i denote i's consumption of good j. Suppose that each consumer i has preferences represented by the utility function

$$U^i(c_1^i, c_2^i) = \alpha_1 lnc_1^i + \alpha_2 lnc_2^i$$

where the parameters α_j are positive, and independent of i, with $\alpha_1 + \alpha_2 = 1$. Suppose the goods are to be distributed in order to maximize social welfare in the form of the weighted sum $W = \beta_1 U^1 + \beta_2 U^2$, where the weights β_i are positive, and $\beta_1 + \beta_2 = 1$.

a) Formulate the welfare maximization problem.

b) Write down the Lagrangian, where λ_j denotes the Lagrange multiplier associated with the constraint for good j. Find the welfare maximizing distribution of the goods. Interpret the value of the Lagrange multiplier λ_j.

c) Verify that $\lambda_j = (\partial W^*)/(\partial e_j)$, where W^* denotes the maximum value of W.

2) Consumer Theory II

A household maximizes its utility from consuming electricity (E) and a bundle of other consumption goods (X) each month. The utility function can be specified as:

$$U(E, X) = X + \frac{\beta}{\beta+1} E^{\frac{\beta+1}{\beta}}.$$

The price of electricity is p, while the bundled good is a numeraire with its price normalized to 1. The household's monthly income is denoted as M.

a) Calculate the household's Marshallian demand functions for electricity and other consumption goods in terms of prices and income.

b) Determine the household's indirect utility function and derive its expenditure function using duality.

c) Suppose the household invests in a rooftop solar photovoltaic (PV) system with a monthly payment $\bar{C}$. The electricity generated from the solar PV system is denoted as E_{pv}, and it is free. E_{pv} is a perfect substitute for electricity from the grid, denoted as E_g. This means that having the solar PV system reduces the expenditure on electricity. Therefore, $E_{pv} + E_g = E$. Once installed, E_{pv} is a fixed amount. Assuming that $E_{pv} < E$, construct the new budget constraint for the household.

d) How would the utility-maximizing levels of E and X change for the household after installing the solar PV system? Solve for the new Marshallian demand functions in terms of prices, income, and E_{pv}. Comment on the result in comparison to your findings in a).

e) Assuming $\bar{C} = 0$, illustrate the analysis above in a graph of the decision making before and after investing in the solar PV system. Show the budget constraints, the indifference curves, and the Marshallian demand. Clearly label the axes and the lines. Use the graph to comment on how the household demand bundles would change if E_{pv} increases.

3) Game Theory

Consider the following international trade game in which there are three types of agents: (1) domestic firms, (2) foreign firms, and (3) a domestic government. In this game the government moves first setting a per-unit tariff (which is represented by t) that is charged to each foreign firm. After the domestic government has chosen the per unit tariff rate, the domestic and foreign firms move at the same time, choosing how much quantity to produce. In this game there are n_f foreign firms, n_d domestic firms, and the industry has an inverse demand curve equal to $P(Q) = \propto - Q$. Firms are profit maximizers and each firm has no fixed costs. The marginal cost for the domestic firm is represented by c_d, the marginal cost for the foreign firm is represented by c_f. The government objective is to maximize social welfare, which is equal to $W = .5Q^2 + n_d\pi_d + n_d q_f * t$.

a) First, assume that there is one foreign firm ($n_f = 1$) and no domestic firms ($n_d = 0$). What quantity will the foreign firm produce? What will the tariff be? What will be the domestic social welfare in this industry?

b) Next, assume that there are no foreign firms ($n_f = 0$) and one domestic firm ($n_d = 1$). What quantity will the domestic firm produce? What will be the domestic social welfare be in this industry?

c) Next, assume $\alpha = 100$, $c_d = 40$ and $c_f = 20$. Calculate the domestic welfare under each 2 above scenarios (part a and part b). Which scenario provides more domestic welfare? What would the domestic social welfare be if the domestic government was not allowed to charge a tariff?

d) Finally, assume $n_f = 1$, $n_d = 1$. and $c_d = 2c_f$. What will the optimal tariff be? What quantity will each firm produce? What will be the domestic social welfare for this industry?

4) Overlapping Generations with Altruism

Consider an economy consisting of an infinite sequence of two-period lived overlapping generations. Assume time t agents care about their offspring, and leave bequests b_{t+1} (and also received bequests from their parents b_t). In each period there is a single final good that is produced using a constant returns to scale technology with capital and labor as inputs. Let k_t denote the time t capital-labor ratio, and let $f(k_t) = Ak_t^{\alpha}$ denote the intensive production function. Assume no depreciation, and population grows at the rate $n > 0$. Let $\rho > 0$ be the discount factor, and denote the utility of an agent born at time t as V_t, where

$$V_t = \frac{c_{1t}^{1-\theta} - 1}{1-\theta} + (1+\rho)^{-1} \left[\frac{c_{2t+1}^{1-\theta} - 1}{1-\theta} + (1+n)V_{t+1} \right]$$

a) Solve V_t recursively forward. What does your solution look like?

b) Derive the household's maximization problem and the first-order conditions. Carefully discuss.

c) Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form.

d) What are the equilibrium conditions for this economy?

e) Now combine your results in b)-d) to get a Law of Motion equation. What does the *LoM* look like? Is this model Pareto optimal? Why/Why not? Carefully show why and discuss.